A sequence is an ordered list of numbers. 
The sum of the terms of a sequence is called a series.

While some sequences are simply random values, other sequences have a definite pattern that is used to arrive at the sequence's terms. Two such sequences are the arithmetic and geometric sequences. Let's investigate the geometric sequence.

**Geometric Sequences**

If a sequence of values follows a pattern of multiplying a fixed amount (not zero) times each term to arrive at the following term, it is referred to as a geometric sequence. The number multiplied each time is constant (always the same).

The fixed amount multiplied is called the common ratio, \( r \), referring to the fact that the ratio (fraction) of the second term to the first term yields this common multiple. To find the common ratio, divide the second term by the first term.

Notice the non-linear nature of the scatter plot of the terms of a geometric sequence. The domain consists of the counting numbers 1, 2, 3, 4, ... and the range consists of the terms of the sequence. While the \( x \) value increases by a constant value of one, the \( y \) value increases by multiples of two (for this graph).

**Examples:**

<table>
<thead>
<tr>
<th>Geometric Sequence</th>
<th>Common Ratio, ( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 10, 20, 40, ...</td>
<td>( r = 2 ) multiply each term by 2 to arrive at the next term or...divide ( a_2 ) by ( a_1 ) to find the common ratio, 2.</td>
</tr>
<tr>
<td>-11, 22, -44, 88, ...</td>
<td>( r = -2 ) multiply each term by -2 to arrive at the next term or...divide ( a_2 ) by ( a_1 ) to find the common ratio, -2.</td>
</tr>
</tbody>
</table>

multiply each term by 2/3 to arrive at the next
Formulas used with geometric sequences and geometric series:

- **To find any term of a geometric sequence:**
  \[ a_{n} = a_{1} \cdot r^{n-1} \]
  where \( a_{1} \) is the first term of the sequence, 
  \( r \) is the common ratio, \( n \) is the number of the term to find.

- **To find the sum of a certain number of terms of a geometric sequence:**
  \[ S_{n} = \frac{a_{1}(1 - r^{n})}{1 - r} \]
  where \( S_{n} \) is the sum of \( n \) terms (\( n^{th} \) partial sum), 
  \( a_{1} \) is the first term, \( r \) is the common ratio.

Note: \( a_{1} \) is often simply referred to as \( a \).

### Examples:

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Find the common ratio for the sequence 6, -3, ( \frac{3}{2} ), -( \frac{3}{4} ), ...</td>
<td>1. The common ratio, ( r ), can be found by dividing the second term by the first term, which in this problem yields -1/2. Checking shows that multiplying each entry by -1/2 yields the next entry.</td>
</tr>
<tr>
<td>2. Find the common ratio for the sequence given by the formula ( a_{n} = 5(3)^{n-1} )</td>
<td>2. The formula indicates that 3 is the common ratio by its position in the formula. A listing of the terms will also show what is happening in the sequence (start with ( n = 1 )). 5, 15, 45, 135, ... The list also shows the common ratio to be 3.</td>
</tr>
</tbody>
</table>
| 3. Find the 7th term of the sequence 2, 6, 18, 54, ...                     | 3. \( n = 7; \ a_{1} = 2, \ r = 3 \)\n\[ a_{n} = a_{1} \cdot r^{n-1} \]
\[ a_{7} = 2 \cdot 3^{7-1} = 1458 \]
The seventh term is 1458. |
| 4. Find the 11th term of the sequence \( 1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, ... \) | 4. \( n = 11; \ a_{1} = 1, \ r = -1/2 \)\n\[ a_{n} = 1 \cdot \left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1024} \] |
| 5. Find \( a_{8} \) for the sequence 0.5, 3.5, 24.5, 171.5, ...           | 5. \( n = 8; \ a_{1} = 0.5, \ r = 7 \) |
6. Evaluate using a formula:

\[ \sum_{k=1}^{5} 3^k \]

\[ \alpha_n = \alpha_1 \cdot r^{n-1} \]
\[ \alpha_2 = 0.5 \cdot 7^{2-1} = 411,771.5 \]

6. Examine the summation

\[ \sum_{k=1}^{5} 3^k = 3^1 + 3^2 + 3^3 + 3^4 + 3^5 \]

This is a geometric series with a common ratio of 3.

\[ n = 5; \quad a_1 = 3, \quad r = 3 \]

\[ S_n = \frac{3(1 - 3^5)}{1 - 3} = \frac{-726}{-2} = 363 \]

7. Find the sum of the first 8 terms of the sequence

-5, 15, -45, 135, ...

7. The word "sum" indicates a need for the sum formula.

\[ n = 8; \quad a_1 = -5, \quad r = -3 \]

\[ S_n = \frac{-5(1 - (-3)^8)}{1 - (-3)} \]
\[ S_8 = \frac{-5(1 - 6561)}{4} = \frac{32800}{4} = 8200 \]

8. The third term of a geometric sequence is 3 and the sixth term is 1/9. Find the first term.

8. Think of the sequence as "starting with" 3, until you find the common ratio.

\[ \_\_\_, \_\_\_, \_\_\_, 3, \_\_\_, \_\_\_, \_\_\_, \frac{1}{9} \]

For this modified sequence: \( a_1 = 3, \ a_4 = 1/9, \ n = 4 \)

\[ a_n = a_1 \cdot r^{n-1} \]
\[ \frac{1}{9} = 3 \cdot r^{4-1} \]
\[ \frac{1}{27} = r^3 \]
\[ \frac{1}{3} = r \]

Now, work backward multiplying by 3 (or dividing by 1/3) to find the actual first term.

\( a_1 = 27 \)

9. A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. How high (to the nearest tenth of a foot) does the ball bounce on the fifth bounce?

9. Set up a model drawing for each "bounce".

\[ 6.4, 5.12, \_\_\_, \_\_\_, \_\_\_, \_\_\_ \]

The common ratio is 0.8.

\[ a_n = a_1 \cdot r^{n-1} \]
\[ a_n = 6.4 \cdot (0.8)^{5-1} = 2.62144 \]
Answer: 2.6 feet

Check out how to use your TI-83+/84+ graphing calculator with sequences and series. Click here.

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