A sequence is an ordered list of numbers. The sum of the terms of a sequence is called a series.

While some sequences are simply random values, other sequences have a definite pattern that is used to arrive at the sequence's terms. Two such sequences are the arithmetic and geometric sequences. Let's investigate the arithmetic sequence.

If a sequence of values follows a pattern of adding a fixed amount from one term to the next, it is referred to as an arithmetic sequence. The number added to each term is constant (always the same).

The fixed amount is called the common difference, \(d\), referring to the fact that the difference between two successive terms yields the constant value that was added. To find the common difference, subtract the first term from the second term.

Notice the linear nature of the scatter plot of the terms of an arithmetic sequence. The domain consists of the counting numbers 1, 2, 3, 4, ... and the range consists of the terms of the sequence. While the \(x\) value increases by a constant value of one, the \(y\) value increases by a constant value of 3 (for this graph).

**Examples:**

<table>
<thead>
<tr>
<th>Arithmetic Sequence</th>
<th>Common Difference, (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 4, 7, 10, 13, 16, ...</td>
<td>(d = 3) add 3 to each term to arrive at the next term,</td>
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<tr>
<td></td>
<td>or...the difference (a_2 - a_1) is 3.</td>
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<tr>
<td>15, 10, 5, 0, -5, -10, ...</td>
<td>(d = -5) add -5 to each term to arrive at the next term,</td>
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<tr>
<td></td>
<td>or...the difference (a_2 - a_1) is -5.</td>
</tr>
<tr>
<td>1, (\frac{1}{2}), 0, (-\frac{1}{2}), ...</td>
<td>(d = -\frac{1}{2}) add -1/2 to each term to arrive at the</td>
</tr>
<tr>
<td></td>
<td>next term, or...the difference (a_2 - a_1) is -1/2.</td>
</tr>
</tbody>
</table>
Formulas used with arithmetic sequences and arithmetic series:

To find any term of an arithmetic sequence:
\[ a_n = a_1 + (n - 1)d \]
where \( a_1 \) is the first term of the sequence, \( d \) is the common difference, \( n \) is the number of the term to find.

To find the sum of a certain number of terms of an arithmetic sequence:
\[ S_n = \frac{n(a_1 + a_n)}{2} \]
where \( S_n \) is the sum of \( n \) terms (\( n^{th} \) partial sum), \( a_1 \) is the first term, \( a_n \) is the \( n^{th} \) term.

Note: \( a_1 \) is often simply referred to as \( a \).

Examples:

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Find the common difference for this arithmetic sequence 5, 9, 13, 17...</td>
<td>1. The common difference, ( d ), can be found by subtracting the first term from the second term, which in this problem yields 4. Checking shows that 4 is the difference between all of the entries.</td>
</tr>
</tbody>
</table>
| 2. Find the common difference for the arithmetic sequence whose formula is \( a_n = 6n + 3 \) | 2. The formula indicates that 6 is the value being added (with increasing multiples) as the terms increase. A listing of the terms will also show what is happening in the sequence (start with \( n = 1 \)). 9, 15, 21, 27, 33, ...
The list shows the common difference to be 6. |
| 3. Find the 10\(^{th}\) term of the sequence 3, 5, 7, 9, ...              | 3. \( n = 10; \ a_1 = 3, \ d = 2 \) \[ a_n = a_1 + (n-1)d \] \[ a_{10} = 3 + (10-1)2 \] \[ a_{10} = 21 \] The tenth term is 21. |
| 4. Find \( a_7 \) for an arithmetic sequence where \( a_1 = 3x \) and \( d = -x \). | 4. \( n = 7; \ a_1 = 3x, \ d = -x \) \[ a_n = a_1 + (n-1)d \] \[ a_7 = 3x + (7-1)(-x) \] \[ a_7 = 3x + 6(-x) = -3x \] |
| 5. Find \( t_{15} \) for an arithmetic sequence where \( t_3 = -4 + 5i \) and \( t_6 = -13 + 11i \) | 5. Notice the change of labeling from \( a \) to \( t \). The letter used in labeling is of no importance. Get a visual image of this problem |
Using the third term as the "first" term, find the common difference from these known terms.

\[ a_n = a_1 + (n - 1)d \]
\[ t_6 = t_3 + (4 - 1)d \]
\[ -13 + 11i = -4 + 5i + (4 - 1)d \]
\[ -13 + 11i = -4 + 5i + 3d \]
\[ -9 + 6i = 3d \]
\[ -3 + 2i = d \]
Now, from \( t_3 \) to \( t_{15} \) is 13 terms.
\[ t_{15} = -4 + 5i + (13 - 1)(-3 + 2i) = -4 + 5i - 36 + 24i \]
\[ = -40 + 29i \]

6. Find a formula for the sequence 1, 3, 5, 7, ...

6. A formula will relate the subscript number of each term to the actual value of the term.
\[ a_n = 2n - 1 \]
Substituting \( n = 1 \), gives 1.
Substituting \( n = 2 \), gives 3, and so on.

7. Find the 25th term of the sequence -7, -4, -1, 2, ...

7. \( n = 25; \ a_1 = -7, \ d = 3 \)
\[ a_n = a_1 + (n - 1)d \]
\[ a_{25} = -7 + (25 - 1)3 \]
\[ a_{25} = 65 \]

8. Find the sum of the first 12 positive even integers.

8. The word "sum" indicates the need for the sum formula.

positive even integers: 2, 4, 6, 8, ...
\( n = 12; \ a_1 = 2, \ d = 2 \)
We are missing \( a_{12} \), for the sum formula, so we use the "any term" formula to find it.
\[ a_n = a_1 + (n - 1)d \]
\[ a_{12} = 2 + (12 - 1)2 \]
\[ a_{12} = 24 \]
Now, let's find the sum:
\[ S_{12} = \frac{12(2 + 24)}{2} = 156 \]

9. Insert 3 arithmetic means between 7 and 23.

9. While there are several solution methods, we will use our arithmetic sequence formulas.
Drawing a picture to better understand the situation.
\[ 7, \_\_\_, \_\_\_, \_\_\_, 23 \]
This set of terms will be an arithmetic sequence.
term between any two terms of an arithmetic sequence. It is simply the average (mean) of the given terms.

We know the first term, $a_1$, the last term, $a_n$, but not the common difference, $d$. This question makes NO mention of "sum", so avoid that formula.

Find the common difference:

\[ a_n = a_1 + (n-1)d \]
\[ 23 = 7 + (5-1)d \]
\[ 23 = 7 + 4d \]
\[ 16 = 4d \]
\[ 4 = d \]

Now, insert the terms using $d$.

7, 11, 15, 19, 23

10. Find the number of terms in the sequence 7, 10, 13, ..., 55.

\[ a_1 = 7, \ a_n = 55, \ d = 3. \] We need to find $n$.

This question makes NO mention of "sum", so avoid that formula.

\[ a_n = a_1 + (n-1)d \]
\[ 55 = 7 + (n-1)3 \]
\[ 55 = 7 + 3n - 3 \]
\[ 55 = 4 + 3n \]
\[ 51 = 3n \]
\[ 17 = n \]

When solving for $n$, be sure your answer is a positive integer. There is no such thing as a fractional number of terms in a sequence!

11. A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theater has 20 rows of seats, how many seats are in the theater?

11. The seating pattern is forming an arithmetic sequence. 60, 68, 76, ...

We wish to find "the sum" of all of the seats.

\[ n = 20, \ a_1 = 60, \ d = 8 \] and we need $a_{20}$ for the sum.

\[ a_n = a_1 + (n-1)d \]
\[ a_{20} = 60 + (20-1)8 = 212 \]

Now, use the sum formula:

\[ S_n = \frac{n(a_1+a_n)}{2} \]
\[ S_{20} = \frac{20(60+212)}{2} = 2720 \]

There are 2720 seats.

Check out how to use your TI-83+/84+ graphing calculator with sequences and series. Click here.