Learning to Think Mathematically With the Rekenrek

A Resource for Teachers
A Tool for Young Children

Adapted from the work of
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Overview

- Rekenrek, a simple, but powerful, manipulative to help young children develop mathematical understanding.
  - A rationale for the Rekekrek
  - The mathematics of the Rekenrek
  - Many activities that show how the Rekenrek can improve students understanding and proficiency with addition and subtraction, number sense, and our base-10 system

Introduction

- **Solve the following addition problems mentally.**

8 + 7 = ?

- **What mental adjustments did you make as you solved this problem?**
  - Double 8, subtract 1? (8 + 8 = 16; 16 - 1 = 15)
  - Double 7, add 1? (7 + 7 = 14; 14 + 1 = 15)
  - Make 10, add 5? (8 + 2 = 10; 10 + 5 = 15)
  - Make 10 another way? (7 + 3 = 10; 10 + 5 = 15)
  - Other strategies?

- **Next problem…**

5 + 8 = ?

- **What mental adjustments did you make as you solved this problem?**
  - **Make 10, add 5? (5 + 5 = 10; 10 + 3 = 13)**
  - **Make 10 another way? (8 + 2 = 10; 10 + 3 = 13)**
  - **Use another fact? (If 8 + 4 = 12, then 8 + 5 = 13)**
  - **Other strategies?**

- **Next problem…**

9 + 7 = ?

- **What mental adjustments did you make as you solved this problem?**
  - **Make 10, add 6? (9 + 1 = 10; 10 + 6 = 16)**
  - **Other strategies?**

Question:

- **If we use these strategies as adults, do we teach them explicitly to young children?**
- **Should we?**
- **If so… how?**

What is the Rekenrek?

- The Rekenrek is a powerful tool that helps children see “inside” numbers (“subitize”), develop cardinality (one-to-one correspondence), and work flexibly with numbers by using decomposition strategies.

Suggested Introduction

- Give your students time to explore the Rekenrek.
  - http://www.brooklynschoolofinquiry.org/blogs/nnelson/post/the-rekenrek-a-math-tool

- Provide students with opportunities to:
  - Show me 1 – 10 (look for non-counting strategies!)
  - Make 5; Make 10 (with two rows)
  - Emphasize completing these steps with “one push”

- More challenging activities.

Flash Attack

- Objectives
  - Subitizing
  - Visual anchors around 5 and 10
  - To help students make associations between various quantities. For example, consider the way a child might make the connection between 8 and 10.
    - *“I know there are ten beads in each row. There were two beads left in the start position. So… there must be 8 in the row because 10 - 2 = 8.”*

Lesson Progression

- Start with the top row only; Push over 2 beads: “How many do you see?”
- Repeat: push over 4 beads; 5 beads
- Now… instruct students that they have only two seconds to tell how many beads are visible.
- Suggested sequence:
  - 6, 10, 9, 7, 8, 5, 3, 4
- Ready?

See and Slide

- Choose a number (1-10) at random (popsicle sticks, dice)
- Challenge children to move in one slide.
- Share
- Add numerals to 20.

Basic Combinations 0-10

- Our children are only used to seeing problems like…
  - » 4 + 5 = ? 6 + 3 = ?

- Doubles and Near Doubles
- Part-Part-Whole relationships
- Missing Addend problems
- Continue to build informal strategies and means for combining numbers

Pattern for 7

- 2 reds + 5 whites = 7 beads
- 3 reds + 4 whites = 7 beads
- 4 reds + 3 whites = 7 beads
- 5 reds + 2 whites = 7 beads

Combinations, 10-20

- This activity works the same as the previous, but we use numbers between 10 and 20
- Vary the presentation of the numbers, using context occasionally

- Example…
  - “Let’s make 15. I start with 8. How many more?”
  - Think through the reasoning used here by the children. What strategies might they use?

Doubles

- Objectives
  - Help students visualize doubles (e.g., 4+4; 6+6)
  - Help students use doubles in computation
- The visualization is key!

Doubles with Your Students

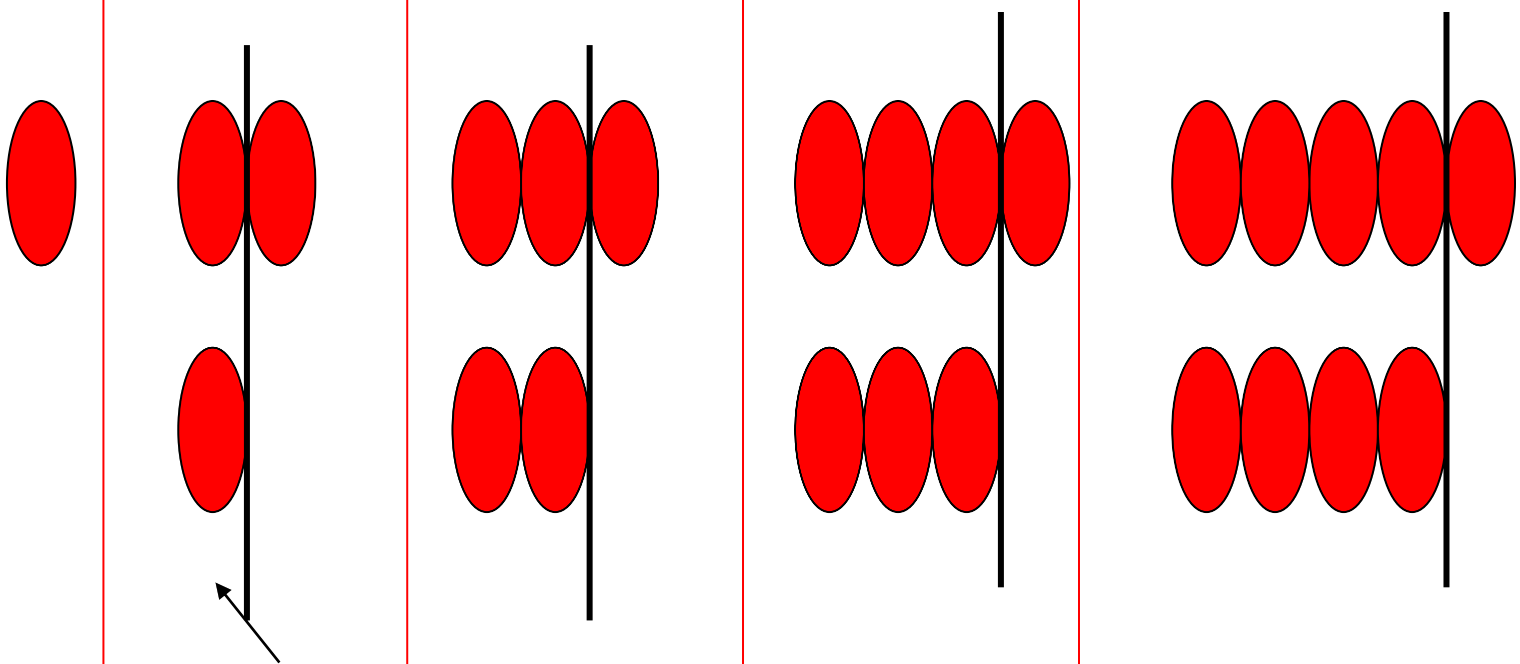
- Show a number with a tens frame card.
- What do you see?
- Can you build it as a double?
- After several building with equal beads, show two different even numbers. See what happens! Challenge them to build those two combined as a double.

Almost a Double

- Students should use their understanding of doubles to successfully work with “near doubles”, e.g., 7 + 8
- Show an odd tens frame
- Students can begin to recognize the difference between even numbers (even numbers can be represented as a pair of equal numbers) and odd numbers (paired numbers plus one).

Near Doubles…

	2 + 1	3 + 2	4 + 3	5+ 4
1	2+1 = 3	4+1 = 5	6+1 = 7	8+1 = 9



Use a pencil to separate

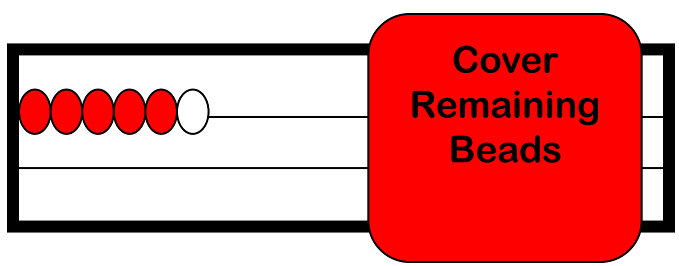
Near Doubles: Developing Ideas

- Develop this idea by doing several additional examples with the Rekenrek. Ask students to use the Rekenrek to "prove" whether or not the following are true.
- Have students visually identify each component of these statements:
  - Does 6 + 7 = 12 + 1?
  - Does 3 + 2 = 4 + 1?
  - Does 4 + 5 = 8 + 1?
  - Does 8 + 9 = 16 + 1?

Part-Part-Whole

- **Push some beads to the left. Cover the remaining beads.**

- **Ask students: "How many beads do you see on the top row?" "How many beads are covered (top row)?"**



It's Automatic: Math Facts

- **Quick recall, yes. But… with understanding, and with a strategy!**
  - anchoring on 10 and
  - using doubles
  - Students can model the number facts on one row of the rekenrek (like 5 + 4), or model facts using both rows (which they have to do when the number gets larger, e.g., 8 + 7)
  - Use flashcards with rekenrek image on the front and number sentence on the back.

Subtraction Activities

- Target Think Addition (page 20)
- Tens and Ones
- Count and Compare

In the Long Run…

- The process of generalization can (should) begin in the primary grades
- Take, for example, the = sign

What belongs in the box?

$$8 + 4 = \square + 5$$

How do children often answer this problem? Discuss…

$8 + 4 = \square + 5$

- 3 research studies used this exact problem
- No more than 10% of US students in grades 1-6 in these 3 studies put the correct number (7) in the box. In one of the studies, not one 6th grader out of 145 put a 7 in the box.
- The most common responses?
- 12 and 17
- Why?
- Students are led to believe through basic fact exercises that the “problem” is on the left side, and the “answer” comes after the = sign.
- Rekenrek use in K-3 minimizes this misconception

- Rather than viewing the = sign as the button on a calculator that gives you the answer, children must view the = sign as a symbol that highlights a *relationship* in our number system.
- For example, how do you "do" the left side of this equation? What work can you possibly do to move toward an answer?

$$3x + 5 = 20$$

- We can take advantage of this "relational" idea in teaching basic facts, manipulating operations, and expressing generalizations in arithmetic at the earliest grades
- For example…
  - 9 = 8 + 1
  - 5 x 6 = (5 x 5) + 5
  - (2 + 3) x 5 = (2 x 5) + (3 x 5)

In Summary…

- Ideas, Insights, Questions about the Rekenrek?
- How it can be used to foster number sense?
- Strengths?
- Limitations?

Constructing a Rekenrek

- What do we need?
  - A small cardboard rectangle (foam board)
  - String/Pipe cleaners
  - 20 beads (10 red, 10 white)

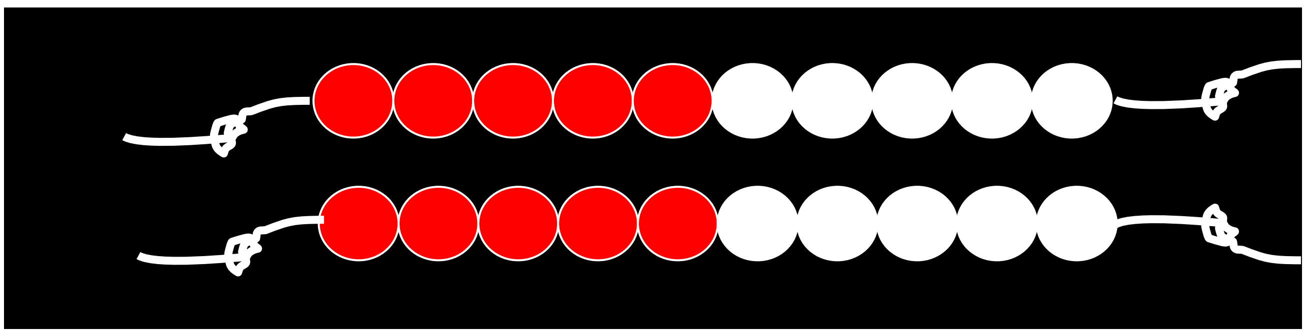
  - For younger children, one string with 10 beads may be sufficient.

How to Make a Rekenrek

Step one: Cut 4 small slits in the cardboard

Step 2: String beads

- 20 beads 10 Red, 10 White
- Two strings of 10 beads



- Tie a knot in the end of each string, or use pipe cleaners with snugly fitting beads.

Step 3: Strings on Cardboard

- **Slip the ends of the string through the slits on the cardboard so that the beads are on the front of the cardboard, and the knot of the string is on the back side.**