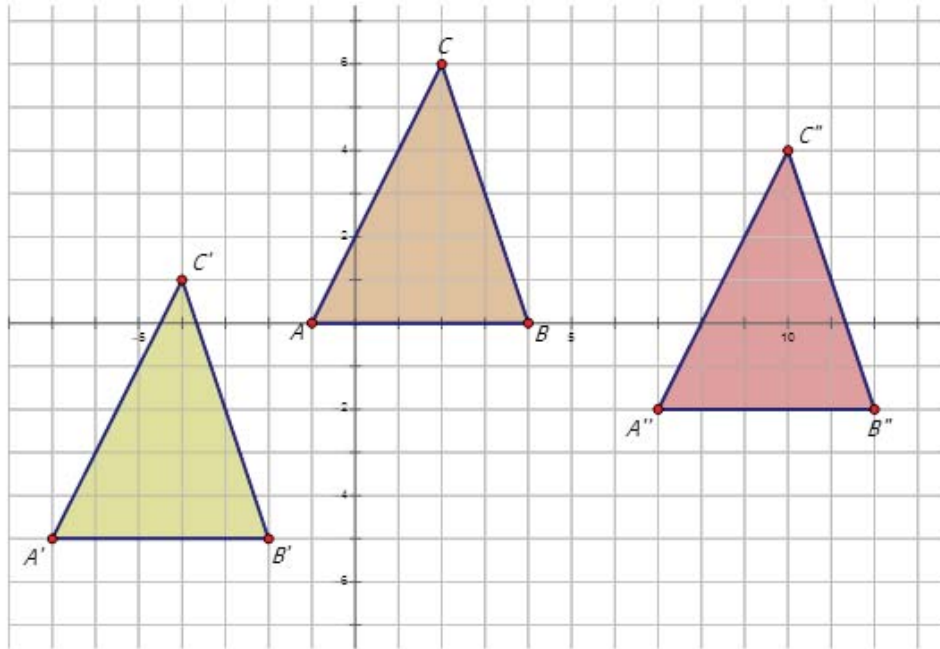


## 1.10 Composite Transformations

Here you will learn about composite transformations.

Look at the following diagram. It involves two translations. Identify the two translations of triangle  $ABC$ .



### Watch This

First watch this video to learn about composite transformations.



#### MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10CompositeTransformationsA](#)

Then watch this video to see some examples.



#### MEDIA

Click image to the left for more content.

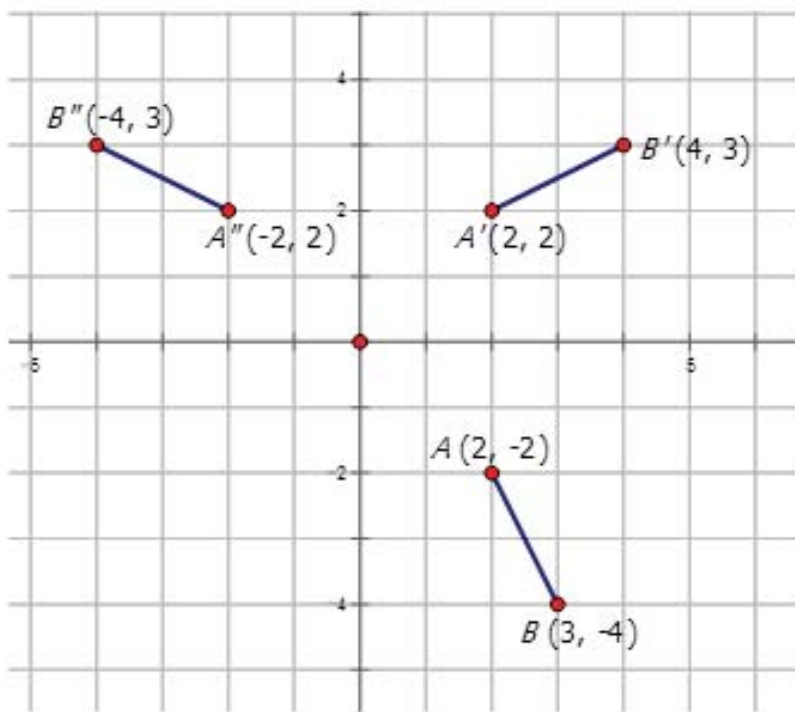
[CK-12 FoundationChapter10CompositeTransformationsB](#)

### Guidance

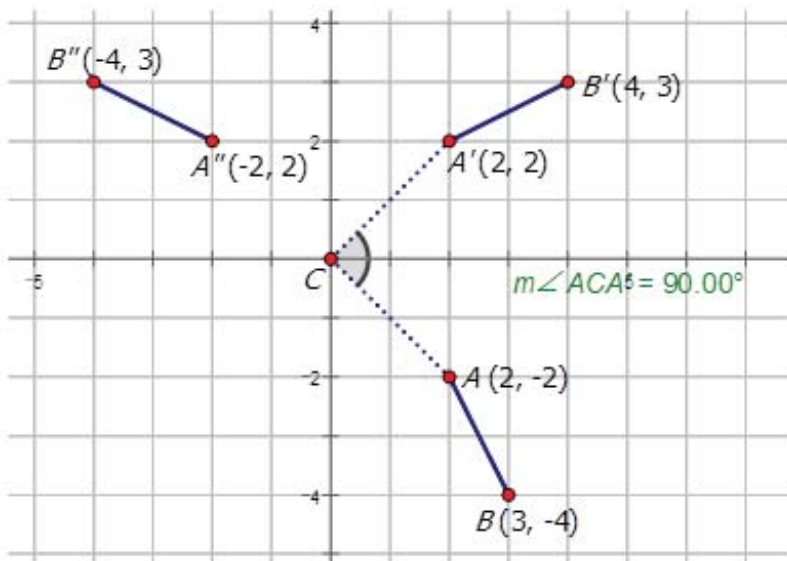
In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image).

### Example A

Describe the transformations in the diagram below. The transformations involve a reflection and a rotation.



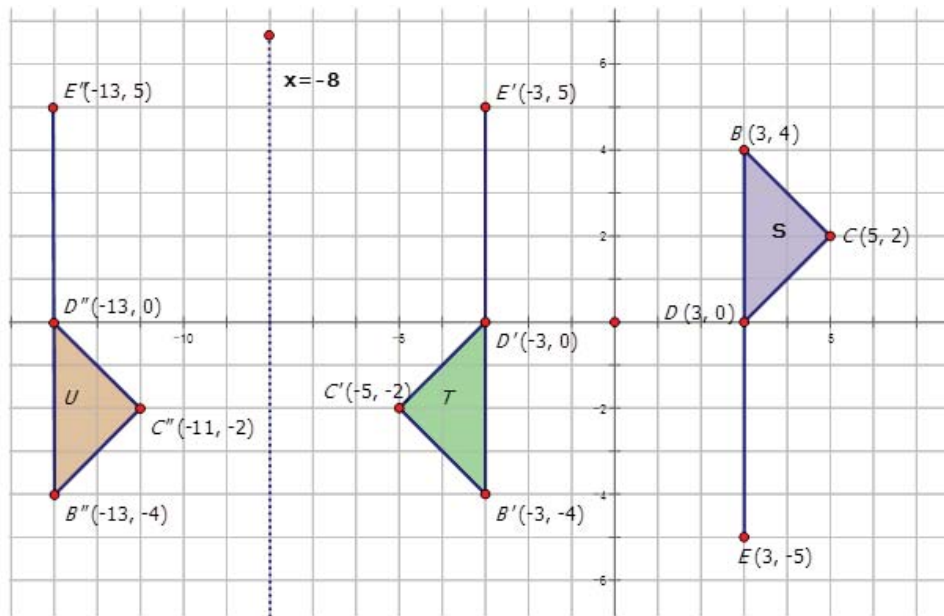
**Solution:** First line  $AB$  is rotated about the origin by  $90^\circ$  CCW.



Then the line  $A'B'$  is reflected about the  $y$ -axis to produce line  $A''B''$ .

**Example B**

Describe the transformations in the diagram below.



**Solution:** The flag in diagram S is rotated about the origin  $180^\circ$  to produce flag T. You know this because if you look at one point you notice that both  $x$ - and  $y$ -coordinate points is multiplied by  $-1$  which is consistent with a  $180^\circ$  rotation about the origin. Flag T is then reflected about the line  $x = -8$  to produce Flag U.

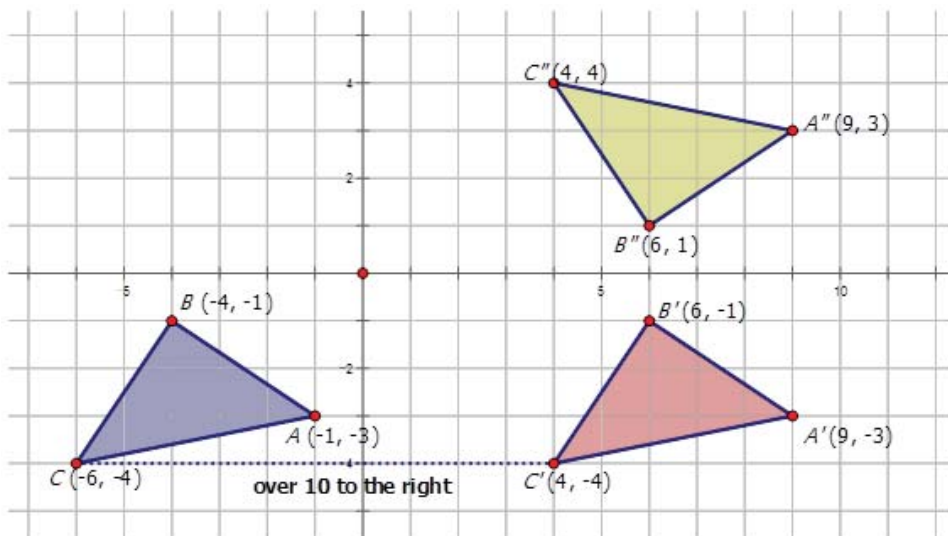
**Example C**

Triangle  $ABC$  where the vertices of  $\triangle ABC$  are  $A(-1, -3)$ ,  $B(-4, -1)$ , and  $C(-6, -4)$  undergoes a composition of transformations described as:

- a translation 10 units to the right, then
- a reflection in the  $x$ -axis.

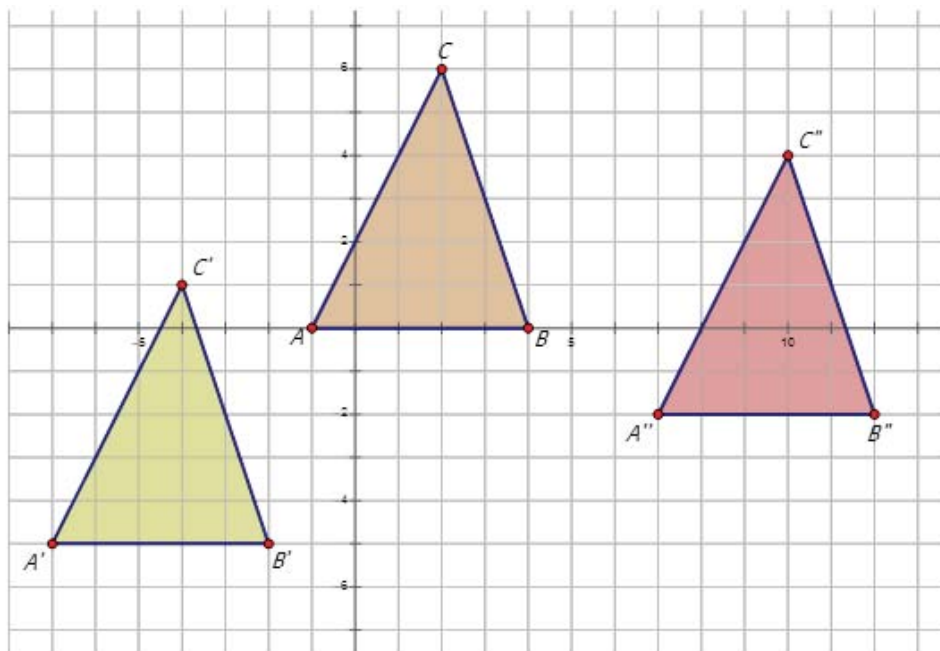
Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied?

**Solution:**

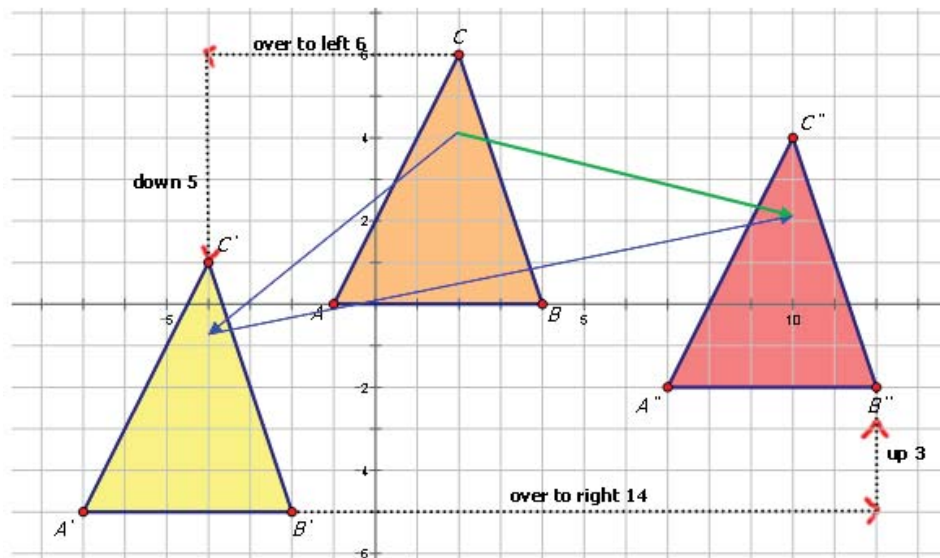


Triangle  $A''B''C''$  is the final triangle after all transformations are applied. It has vertices of  $A''(9, 3)$ ,  $B''(6, 1)$ , and  $C''(4, 4)$ .

**Concept Problem Revisited**



$\triangle ABC$  moves over 6 to the left and down 5 to produce  $\triangle A'B'C'$ . Then  $\triangle A'B'C'$  moves over 14 to the right and up 3 to produce  $\triangle A''B''C''$ . These translations are represented by the blue arrows in the diagram.



All together  $\triangle ABC$  moves over 8 to the right and down 2 to produce  $\triangle A''B''C''$ . The total translations for this movement are seen by the green arrow in the diagram above.

## Vocabulary

### Image

In a transformation, the final figure is called the *image*.

### Preimage

In a transformation, the original figure is called the *preimage*.

### Transformation

A *transformation* is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

### Dilation

A *dilation* is a transformation that enlarges or reduces the size of a figure.

### Translation

A *translation* is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides**.

### Rotation

A *rotation* is a transformation that rotates (turns) an image a certain amount about a certain point.

### Reflection

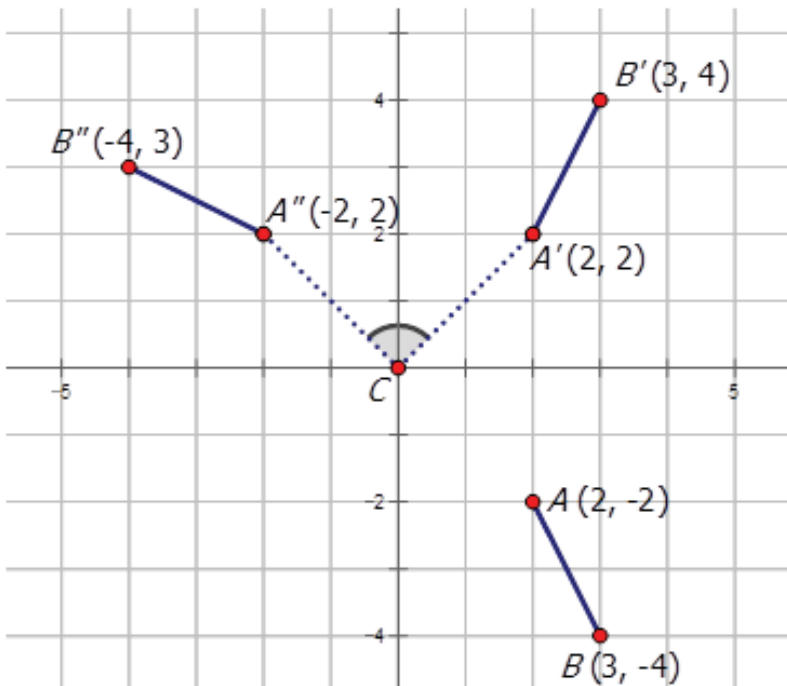
A *reflection* is an example of a transformation that flips each point of a shape over the same line.

### Composite Transformation

A *composite transformation* is when two or more transformations are combined to form a new image from the preimage.

**Guided Practice**

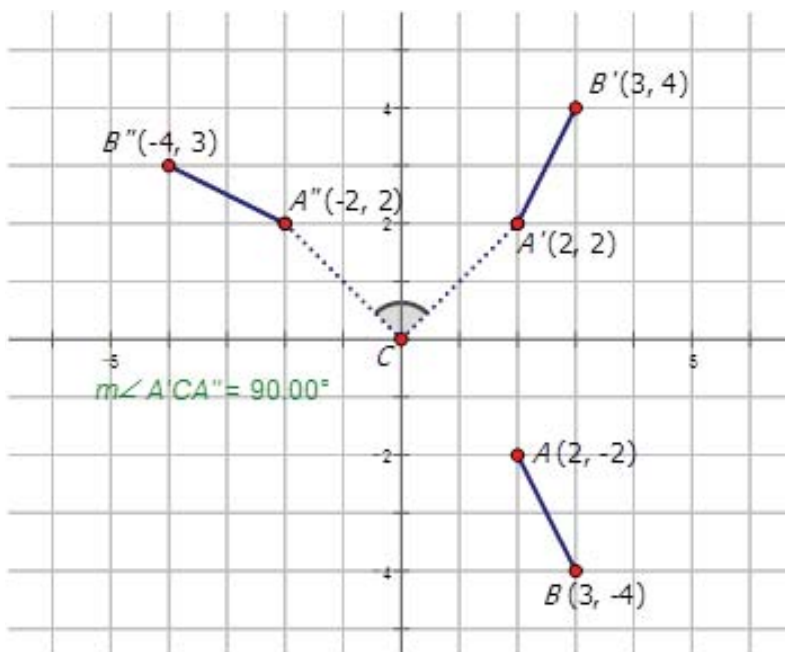
1. Describe the transformations in the diagram below. The transformations involve a rotation and a reflection.



2. Triangle  $XYZ$  has coordinates  $X(1,2)$ ,  $Y(-3,6)$  and  $Z(4,5)$ . The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle  $X'Y'Z'$ . Triangle  $X'Y'Z'$  is then reflected about the  $y$ -axis to form triangle  $X''Y''Z''$ . Draw the diagram of this composite transformation and determine the vertices for triangle  $X''Y''Z''$ .
3. The coordinates of the vertices of  $\Delta JAK$  are  $J(1,6)$ ,  $B(2,9)$ , and  $C(7,10)$ .
  - a) Draw and label  $\Delta JAK$ .
  - b)  $\Delta JAK$  is reflected over the line  $y = x$ . Graph and state the coordinates of  $\Delta J'A'K'$ .
  - c)  $\Delta J'A'K'$  is then reflected about the  $x$ -axis. Graph and state the coordinates of  $\Delta J''A''K''$ .
  - d)  $\Delta J''A''K''$  undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of  $\Delta J'''A'''K'''$ .

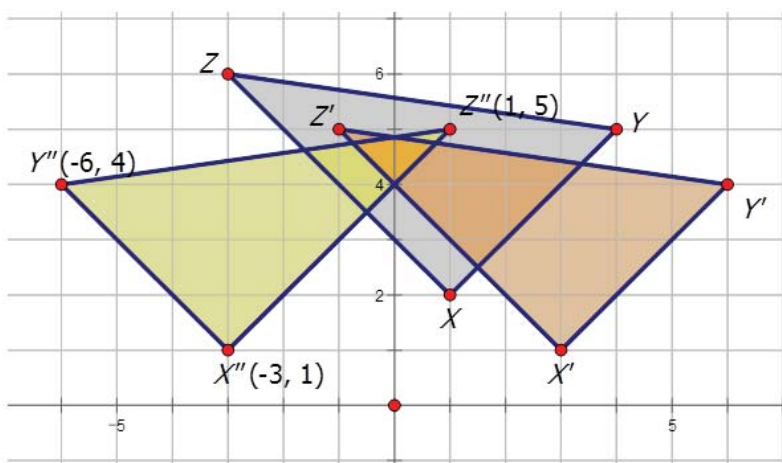
**Answers:**

1. The transformations involve a reflection and a rotation. First line  $AB$  is reflected about the  $y$ -axis to produce line  $A'B'$ .

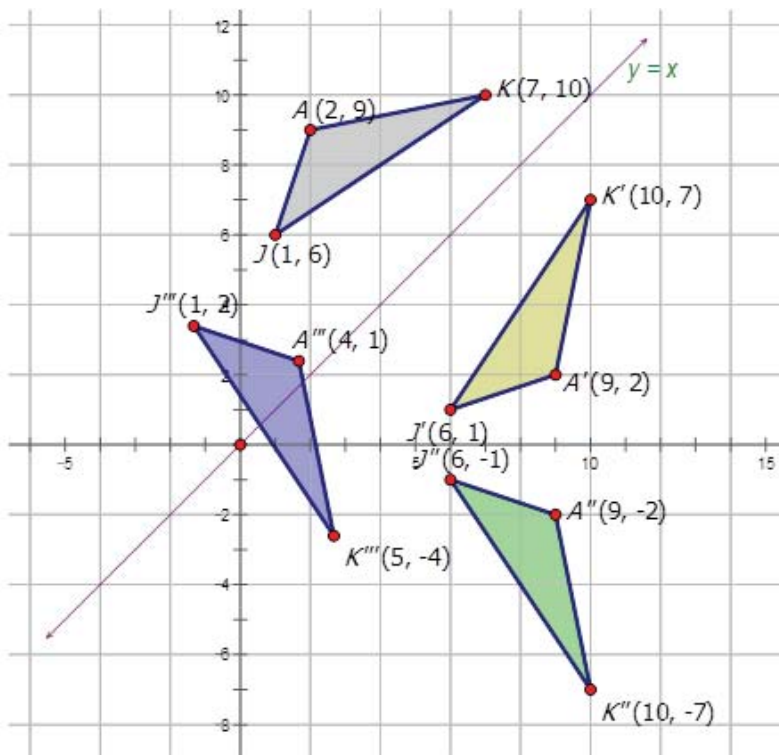


Then the line  $A'B'$  is rotated about the origin by  $90^\circ$  CCW to produce line  $A''B''$ .

2.



3.

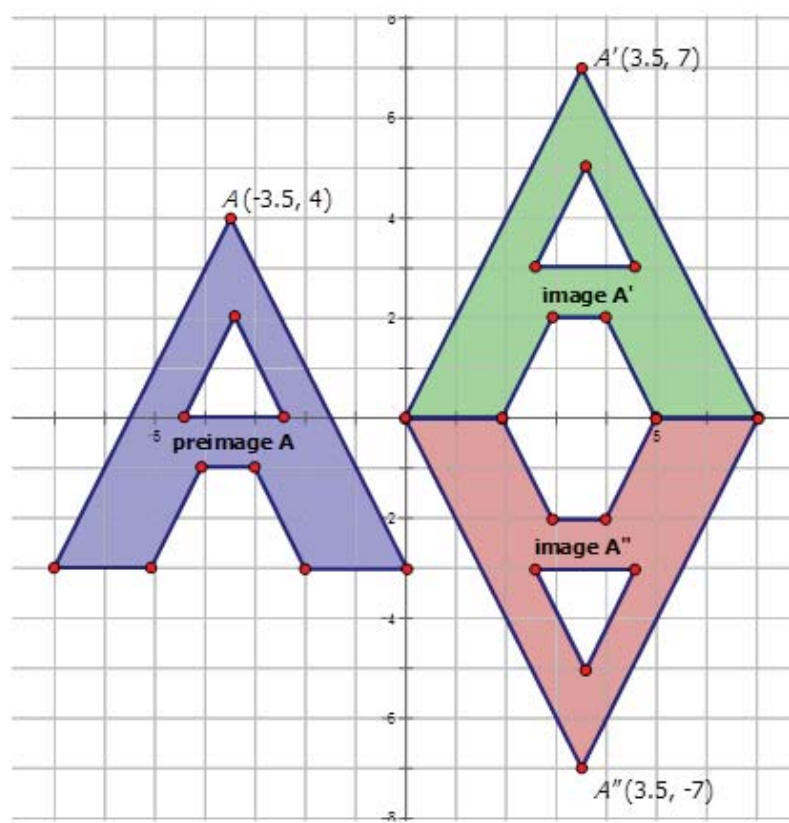
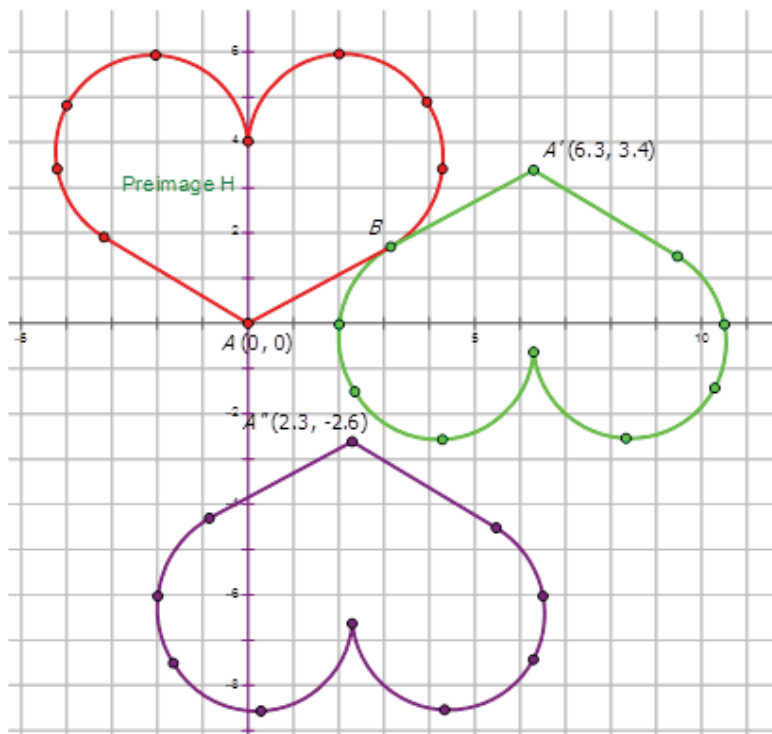


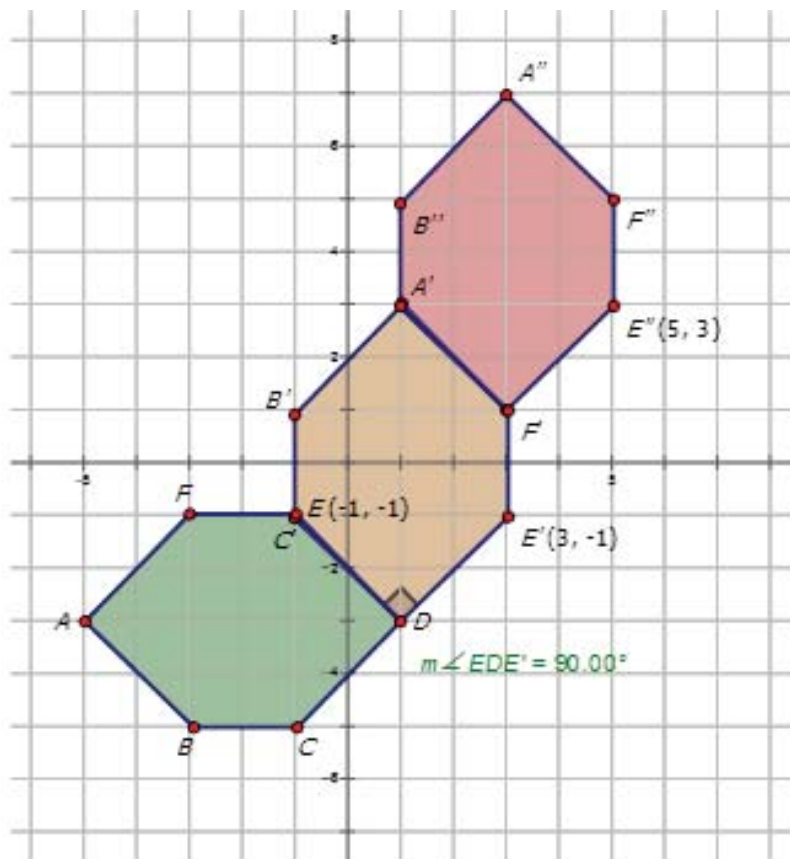
**Practice**

1. A point  $X$  has coordinates  $(-1, -8)$ . The point is reflected across the  $y$ -axis to form  $X'$ .  $X'$  is translated over 4 to the right and up 6 to form  $X''$ . What are the coordinates of  $X'$  and  $X''$ ?
2. A point  $A$  has coordinates  $(2, -3)$ . The point is translated over 3 to the left and up 5 to form  $A'$ .  $A'$  is reflected across the  $x$ -axis to form  $A''$ . What are the coordinates of  $A'$  and  $A''$ ?
3. A point  $P$  has coordinates  $(5, -6)$ . The point is reflected across the line  $y = -x$  to form  $P'$ .  $P'$  is rotated about the origin  $90^\circ$  CW to form  $P''$ . What are the coordinates of  $P'$  and  $P''$ ?
4. Line  $JT$  has coordinates  $J(-2, -5)$  and  $T(2, 3)$ . The segment is rotated about the origin  $180^\circ$  to form  $J'T'$ .  $J'T'$  is translated over 6 to the right and down 3 to form  $J''T''$ . What are the coordinates of  $J'T'$  and  $J''T''$ ?
5. Line  $SK$  has coordinates  $S(-1, -8)$  and  $K(1, 2)$ . The segment is translated over 3 to the right and up 3 to form  $S'K'$ .  $S'K'$  is rotated about the origin  $90^\circ$  CCW to form  $S''K''$ . What are the coordinates of  $S'K'$  and  $S''K''$ ?
6. A point  $K$  has coordinates  $(-1, 4)$ . The point is reflected across the line  $y = x$  to form  $K'$ .  $K'$  is rotated about the origin  $270^\circ$  CW to form  $K''$ . What are the coordinates of  $K'$  and  $K''$ ?

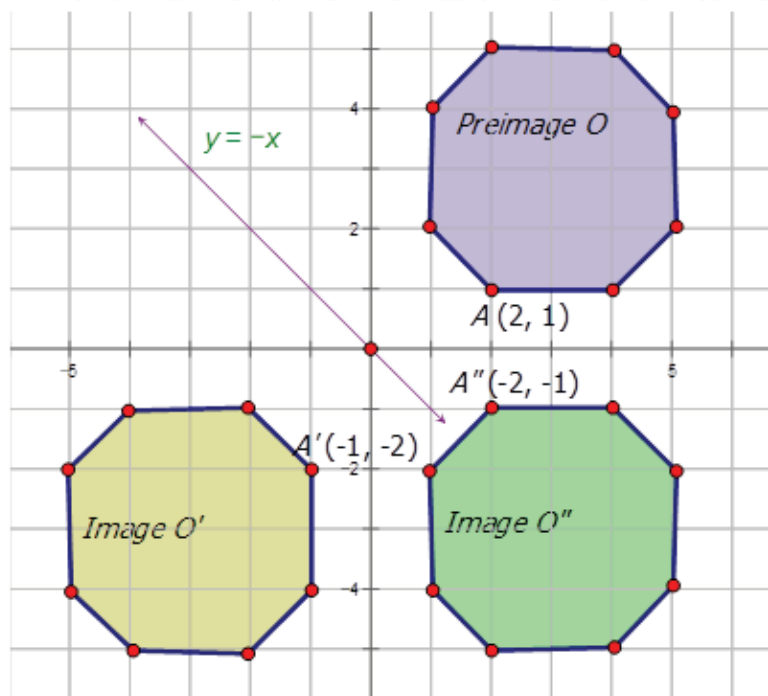
Describe the following composite transformations:



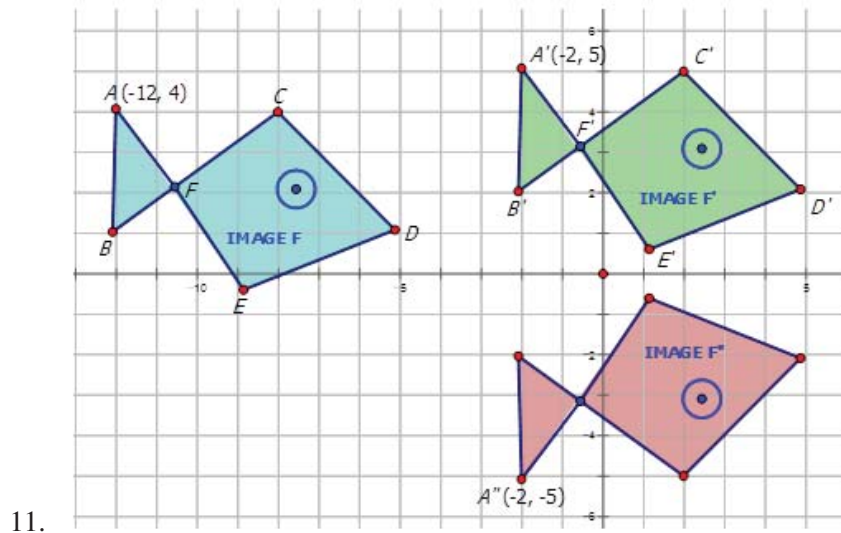




9.



10.



12. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?
13. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?
14. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?
15. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it's called a **glide** reflection?