1.10 Composite Transformations

Here you will learn about composite transformations.

Look at the following diagram. It involves two translations. Identify the two translations of triangle $ABC$.

Watch This

First watch this video to learn about composite transformations.

Click image to the left for more content.

CK-12 FoundationChapter10CompositeTransformationsA

Then watch this video to see some examples.

Click image to the left for more content.

CK-12 FoundationChapter10CompositeTransformationsB
Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image).

Example A

Describe the transformations in the diagram below. The transformations involve a reflection and a rotation.

Solution: First line $AB$ is rotated about the origin by $90^\circ$ CCW.

Then the line $A'B'$ is reflected about the $y$-axis to produce line $A''B''$. 
Example B

Describe the transformations in the diagram below.

Solution: The flag in diagram S is rotated about the origin $180^\circ$ to produce flag T. You know this because if you look at one point you notice that both $x$- and $y$-coordinate points is multiplied by -1 which is consistent with a $180^\circ$ rotation about the origin. Flag T is then reflected about the line $x = -8$ to produce Flag U.

Example C

Triangle $ABC$ where the vertices of $\triangle ABC$ are $A(-1, -3)$, $B(-4, -1)$, and $C(-6, -4)$ undergoes a composition of transformations described as:

a) a translation 10 units to the right, then

b) a reflection in the $x$-axis.

Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied?

Solution:
Triangle $A''B''C''$ is the final triangle after all transformations are applied. It has vertices of $A''(9,3)$, $B''(6,1)$, and $C''(4,4)$.

**Concept Problem Revisited**

$\triangle ABC$ moves over 6 to the left and down 5 to produce $\triangle A'B'C'$. Then $\triangle A'B'C'$ moves over 14 to the right and up 3 to produce $\triangle A''B''C''$. These translations are represented by the blue arrows in the diagram.
All together $\triangle ABC$ moves over 8 to the right and down 2 to produce $\triangle A''B''C''$. The total translations for this movement are seen by the green arrow in the diagram above.

**Vocabulary**

**Image**
In a transformation, the final figure is called the *image*.

**Preimage**
In a transformation, the original figure is called the *preimage*.

**Transformation**
A *transformation* is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

**Dilation**
A *dilation* is a transformation that enlarges or reduces the size of a figure.

**Translation**
A *translation* is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as *slides*.

**Rotation**
A *rotation* is a transformation that rotates (turns) an image a certain amount about a certain point.

**Reflection**
A *reflection* is an example of a transformation that flips each point of a shape over the same line.

**Composite Transformation**
A *composite transformation* is when two or more transformations are combined to form a new image from the preimage.
Guided Practice

1. Describe the transformations in the diagram below. The transformations involve a rotation and a reflection.

2. Triangle $XYZ$ has coordinates $X(1,2)$, $Y(-3,6)$ and $Z(4,5)$. The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle $X'Y'Z'$. Triangle $X'Y'Z'$ is then reflected about the $y$-axis to form triangle $X''Y''Z''$. Draw the diagram of this composite transformation and determine the vertices for triangle $X''Y''Z''$.

3. The coordinates of the vertices of $\Delta JAK$ are $J(1,6)$, $B(2,9)$, and $C(7,10)$.
   a) Draw and label $\Delta JAK$.
   b) $\Delta JAK$ is reflected over the line $y = x$. Graph and state the coordinates of $\Delta J'A'K'$.
   c) $\Delta J'A'K'$ is then reflected about the $x$-axis. Graph and state the coordinates of $\Delta J''A''K''$.
   d) $\Delta J''A''K''$ undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of $\Delta J'''A'''K'''$.

Answers:

1. The transformations involve a reflection and a rotation. First line $AB$ is reflected about the $y$-axis to produce line $A'B'$. 
Then the line $A'B'$ is rotated about the origin by $90^\circ$ CCW to produce line $A''B''$. 

2.

3.
Practice

1. A point \(X\) has coordinates (-1, -8). The point is reflected across the y-axis to form \(X'\). \(X'\) is translated over 4 to the right and up 6 to form \(X''\). What are the coordinates of \(X'\) and \(X''\)?

2. A point \(A\) has coordinates (2, -3). The point is translated over 3 to the left and up 5 to form \(A'\). \(A'\) is reflected across the x-axis to form \(A''\). What are the coordinates of \(A'\) and \(A''\)?

3. A point \(P\) has coordinates (5, -6). The point is reflected across the line \(y = -x\) to form \(P'\). \(P'\) is rotated about the origin 90° CW to form \(P''\). What are the coordinates of \(P'\) and \(P''\)?

4. Line \(JT\) has coordinates \(J(-2, -5)\) and \(T(2, 3)\). The segment is rotated about the origin 180° to form \(J'T'\). \(J'T'\) is translated over 6 to the right and down 3 to form \(J''T''\). What are the coordinates of \(J'T'\) and \(J''T''\)?

5. Line \(SK\) has coordinates \(S(-1, -8)\) and \(K(1, 2)\). The segment is translated over 3 to the right and up 3 to form \(S'K'\). \(S'K'\) is rotated about the origin 90° CCW to form \(S''K''\). What are the coordinates of \(S'K'\) and \(S''K''\)?

6. A point \(K\) has coordinates (-1, 4). The point is reflected across the line \(y = x\) to form \(K'\). \(K'\) is rotated about the origin 270° CW to form \(K''\). What are the coordinates of \(K'\) and \(K''\)?

Describe the following composite transformations:
12. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?

13. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?

14. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?

15. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it’s called a **glide** reflection?