What You'll Learn

• You will learn how interference and diffraction patterns demonstrate that light behaves like a wave.

• You will learn how interference and diffraction patterns occur in nature and how they are used.

Why It's Important
Interference and diffraction can be seen all around you. Compact discs demonstrate diffraction, bubbles show interference, and the wings of a *Morpho* butterfly show both.

**Bubble Solution** Bubble solution in a container is transparent. However, if you suspend the solution in a grid of plastic, swirls of color can be seen. These colors are not caused by pigments or dyes in the soap, but by an effect of the wave nature of light.

Think About This ➤
How does bubble solution produce a rainbow of colors?
**19.1 Interference**

In Chapter 16, you learned that light sometimes acts like a wave. Light can be diffracted when it passes by an edge, just like water waves and sound waves can. In Chapters 17 and 18, you learned that reflection and refraction can be explained when light is modeled as a wave. What led scientists to believe that light has wave properties? They discovered that light could be made to interfere, which you will learn about in this section.

When you look at objects that are illuminated by a white light source such as a nearby lightbulb, you are seeing *incoherent light*, which is light with unsynchronized wave fronts. The effect of incoherence in waves can be seen in the example of heavy rain falling on a swimming pool. The surface of the water is choppy and does not have any regular pattern of wave fronts or standing waves. Because light waves have such a high frequency, incoherent light does not appear choppy to you. Instead, as light from an incoherent white light source illuminates an object, you see the superposition of the incoherent light waves as an even, white light.

**Objectives**
- **Explain** how light falling on two slits produces an interference pattern.
- **Calculate** light wavelengths from interference patterns.
- **Apply** modeling techniques to thin-film interference.

**Vocabulary**
- incoherent light
- coherent light
- interference fringes
- monochromatic light
- thin-film interference
Smooth wave fronts of light are created by point sources (a) and by lasers (b).

Interference of Coherent Light

The opposite of incoherent light is coherent light, which is light from two or more sources that add together in superposition to produce smooth wave fronts. A smooth wave front can be created by a point source, as shown in Figure 19-1a. A smooth wave front also can be created by multiple point sources when all point sources are synchronized, such as with a laser, as shown in Figure 19-1b. Only the superposition of light waves from coherent light sources can produce the interference phenomena that you will examine in this section.

English physician Thomas Young proved that light has wave properties when he produced an interference pattern by shining light from a single coherent source through two slits. Young directed coherent light at two closely spaced, narrow slits in a barrier. When the overlapping light from the two slits fell on an observing screen, the overlap did not produce even illumination, but instead created a pattern of bright and dark bands that Young called interference fringes. He explained that these bands must be the result of constructive and destructive interference of light waves from the two slits in the barrier.

In a double-slit interference experiment that uses monochromatic light, which is light of only one wavelength, constructive interference produces a bright central band of the given color on the screen, as well as other bright bands of near-equal spacing and near-equal width on either side, as shown in Figures 19-2a and 19-2b. The intensity of the bright bands decreases the farther the band is from the central band, as you can easily see in Figure 19-2a. Between the bright bands are dark areas where destructive interference occurs. The positions of the constructive and destructive interference bands depend on the wavelength of the light. When white light is used in a double-slit experiment, however, interference causes the appearance of colored spectra instead of bright and dark bands, as shown in Figure 19-2c. All wavelengths interfere constructively in the central bright band, and thus that band is white. The positions of the other colored bands result from the overlap of the interference fringes that occur where wavelengths of each separate color interfere constructively.
Double-slit interference To create coherent light from incoherent light, Young placed a light barrier with a narrow slit in front of a monochromatic light source. Because the width of the slit is very small, only a coherent portion of the light passes through and is diffracted by the slit, producing nearly cylindrical diffracted wave fronts, as shown in Figure 19-3. Because a cylinder is symmetrical, the two portions of the wave front arriving at the second barrier with two slits will be in phase. The two slits at the second barrier then produce coherent, nearly cylindrical wave fronts that can then interfere, as shown in Figure 19-3. Depending on their phase relationship, the two waves can undergo constructive or destructive interference, as shown in Figure 19-4.

Figure 19-3 The coherent source that is created by a narrow single slit produces coherent, nearly cylindrical waves that travel to the two slits in the second barrier. Two coherent, nearly cylindrical waves leave the double slit.

Figure 19-4 A pair of in-phase waves is created at the two slits. At some locations, the waves might undergo constructive interference to create a bright band (a), or destructive interference to create a dark band (b).
Measuring the wavelength of light  A top view of nearly cylindrical wave fronts and Young’s double slit experiment are shown in Figure 19-5a. The wave fronts interfere constructively and destructively to form a pattern of light and dark bands. A typical diagram that is used to analyze Young’s experiment is shown in Figure 19-5b. Light that reaches point $P_0$ travels the same distance from each slit. Because the waves are in phase, they interfere constructively on the screen to create the central bright band at $P_0$. There is also constructive interference at the first bright band, $P_1$, on either side of the central band, because line segment $P_1S_1$ is one wavelength, $\lambda$, longer than the line segment $P_1S_2$. Thus, the waves arrive at $P_1$ in phase.

There are two triangles shaded in the figure. The larger triangle is a right triangle, so $\tan \theta = x/L$. In the smaller triangle $RS_1S_2$, the side $S_1R$ is the length difference of the two light paths, which is one wavelength. There are now two simplifications that make the problem easier to solve.

1. If $L$ is much larger than $d$, then line segments $S_1P_1$ and $S_2P_1$ are nearly parallel to each other and to line segment $QP_1$, and triangle $RS_1S_2$ is very nearly a right triangle. Thus, $\sin \theta = \lambda/d$.

2. If the angle $\theta$ is small, then $\sin \theta$ is very nearly equal to $\tan \theta$.

With the above simplifications, the relationships $\tan \theta = x/L$, $\sin \theta = \lambda/d$, and $\sin \theta = \tan \theta$ combine to form the equation $x/L = \lambda/d$. Solving for $\lambda$ gives the following.

**Wavelength from Double-Slit Experiment**  
\[ \lambda = \frac{xd}{L} \]

The wavelength of light, as measured by a double slit, is equal to the distance on the screen from the central bright band to the first bright band, multiplied by the distance between the slits, divided by the distance to the screen.

Constructive interference from two slits occurs at locations, $x_{m'}$, on either side of the central bright band, which are determined using the equation $m\lambda = x_{m'}d/L$, where $m = 0, 1, 2$, etc., as limited by the small angle simplification. The central bright band occurs at $m = 0$. Frequently, the band given by $m = 1$ is called the first-order band, and so on.
EXAMPLE Problem 1

**Wavelength of Light** A double-slit experiment is performed to measure the wavelength of red light. The slits are 0.0190 mm apart. A screen is placed 0.600 m away, and the first-order bright band is found to be 21.1 mm from the central bright band. What is the wavelength of the red light?

1. **Analyze and Sketch the Problem**
   - Sketch the experiment, showing the slits and the screen.
   - Draw the interference pattern with bands in appropriate locations.

   **Known:**
   - \(d = 1.90 \times 10^{-5}\) m
   - \(x = 2.11 \times 10^{-2}\) m
   - \(L = 0.600\) m

   **Unknown:** \(\lambda = ?\)

2. **Solve for the Unknown**

   \[
   \lambda = \frac{xd}{L} = \frac{(2.11 \times 10^{-2}\) m\}(1.90 \times 10^{-5}) m}{(0.600\) m}\]
   
   \[
   = 6.68 \times 10^{-7}\) nm = 668 nm
   
   Substitute \(x = 2.11 \times 10^{-2}\) m, \(d = 1.90 \times 10^{-5}\) m, \(L = 0.600\) m

3. **Evaluate the Answer**
   - Are the units correct? The answer is in units of length, which is correct for wavelength.
   - Is the magnitude realistic? The wavelength of red light is near 700 nm, and that of blue is near 400 nm. Thus, the answer is reasonable for red light.

**Practice Problems**

1. Violet light falls on two slits separated by \(1.90 \times 10^{-5}\) m. A first-order bright band appears 13.2 mm from the central bright band on a screen 0.600 m from the slits. What is \(\lambda\)?

2. Yellow-orange light from a sodium lamp of wavelength 596 nm is aimed at two slits that are separated by \(1.90 \times 10^{-5}\) m. What is the distance from the central band to the first-order yellow band if the screen is 0.600 m from the slits?

3. In a double-slit experiment, physics students use a laser with \(\lambda = 632.8\) nm. A student places the screen 1.000 m from the slits and finds the first-order bright band 65.5 mm from the central line. What is the slit separation?

4. Yellow-orange light with a wavelength of 596 nm passes through two slits that are separated by \(2.25 \times 10^{-5}\) m and makes an interference pattern on a screen. If the distance from the central line to the first-order yellow band is \(2.00 \times 10^{-2}\) m, how far is the screen from the slits?

Young presented his findings in 1803, but was ridiculed by the scientific community. His conclusions did not begin to gain acceptance until 1820 after Jean Fresnel proposed a mathematical solution for the wave nature of light in a competition. One of the judges, Siméon Denis Poisson, showed that, if Fresnel was correct, a shadow of a circular object illuminated with coherent light would have a bright spot at the center of the shadow. Another judge, Jean Arago, proved this experimentally. Before this, both Poisson and Arago were skeptics of the wave nature of light.
Figure 19-6 Each wavelength is reinforced where the soap film thickness is $\lambda/4$, $3\lambda/4$, $5\lambda/4$ (a). Because each color has a different wavelength, a series of color bands is reflected from the soap film (b).

Thin-Film Interference

Have you ever seen a spectrum of colors produced by a soap bubble or by the oily film on a water puddle in a parking lot? These colors were not the result of separation of white light by a prism or of absorption of colors in a pigment. The spectrum of colors was a result of the constructive and destructive interference of light waves due to reflection in a thin film, a phenomenon called thin-film interference.

If a soap film is held vertically, as in Figure 19-6, its weight makes it thicker at the bottom than at the top. The thickness varies gradually from top to bottom. When a light wave strikes the film, it is partially reflected, as shown by ray 1, and partially transmitted. The reflected and transmitted waves have the same frequency as the original. The transmitted wave travels through the film to the back surface, where, again, part is reflected, as shown by ray 2. This act of splitting each light wave from an incoherent source into a matched pair of waves means that the reflected light from a thin film is coherent.

Color reinforcement How is the reflection of one color enhanced? This happens when the two reflected waves are in phase for a given wavelength. If the thickness of the soap film in Figure 19-6 is one-fourth of the wavelength of the wave in the film, $\lambda/4$, then the round-trip path length in the film is $\lambda/2$. In this case, it would appear that ray 2 would return to the front surface one-half wavelength out of phase with ray 1, and that the two waves would cancel each other based on the superposition principle. However, when a transverse wave is reflected from a medium with a slower wave speed, the wave is inverted. With light, this happens at a medium with a larger index of refraction. As a result, ray 1 is inverted on reflection; whereas ray 2 is reflected from a medium with a smaller index of refraction (air) and is not inverted. Thus, ray 1 and ray 2 are in phase.

If the film thickness, $d$, satisfies the requirement, $d = \lambda/4$, then the color of light with that wavelength will be most strongly reflected. Note that because the wavelength of light in the film is shorter than the wavelength in air, $d = \lambda_{\text{film}}/4$, or, in terms of the wavelength in air, $d = \lambda_{\text{vacuum}}/4n_{\text{film}}$. The two waves reinforce each other as they leave the film. Light with other wavelengths undergoes destructive interference.

Nonreflective Eyeglasses

A thin film can be placed on the lenses of eyeglasses to keep them from reflecting wavelengths of light that are highly visible to the human eye. This prevents the glare of reflected light.
As you know, different colors of light have different wavelengths. For a film of varying thickness, such as the one shown in Figure 19-6, the wavelength requirement will be met at different thicknesses for different colors. The result is a rainbow of color. Where the film is too thin to produce constructive interference for any wavelength of color, the film appears to be black. Notice in Figure 19-6b that the spectrum repeats. When the thickness of the film is \(3\lambda/4\), the round-trip distance is \(3\lambda/2\), and constructive interference occurs again. Any thickness equal to \(1\lambda/4, 3\lambda/4, 5\lambda/4\), and so on, satisfies the conditions for constructive interference for a given wavelength.

**Applications of thin-film interference** The example of a film of soapy water in air involves constructive interference with one of two waves inverted upon reflection. In the chapter opener example of bubble solution, as the thickness of the film changes, the wavelength undergoing constructive interference changes. This creates a shifting spectrum of color on the surface of the film soap when it is under white light. In other examples of thin-film interference, neither wave or both waves might be inverted. You can develop a solution for any problem involving thin-film interference using the following strategies.

### Thin-Film Interference

*When solving thin-film interference problems, construct an equation that is specific to the problem by using the following strategies.*

1. Make a sketch of the thin film and the two coherent waves. For simplicity, draw the waves as rays.

2. Read the problem. Is the reflected light enhanced or reduced? When it is enhanced, the two reflected waves undergo constructive interference. When the reflected light is reduced, the waves undergo destructive interference.

3. Are either or both waves inverted on reflection? If the index of refraction changes from a lower to a higher value, then the wave is inverted. If it changes from a higher to a lower value, there is no inversion.

4. Find the extra distance that the second wave must travel through the thin film to create the needed interference.

   a. If you need constructive interference and one wave is inverted OR you need destructive interference and either both waves or none are inverted, then the difference in distance is an odd number of half wavelengths: \((m + 1/2)\lambda_{\text{film}}\), where \(m = 0, 1, 2, \text{etc.}\)

   b. If you need constructive interference and either both waves or none are inverted OR you need destructive interference and one wave is inverted, then the difference is an integer number of wavelengths: \(m\lambda_{\text{film}}\), where \(m = 1, 2, 3, \text{etc.}\)

5. Set the extra distance traveled by the second ray to twice the film thickness, \(2d\).

6. Recall from Chapter 18 that \(\lambda_{\text{film}} = \lambda_{\text{vacuum}}/n_{\text{film}}\).
5. In the situation in Example Problem 2, what would be the thinnest film that would create a reflected red (λ = 635 nm) band?

6. A glass lens has a nonreflective coating placed on it. If a film of magnesium fluoride, n = 1.38, is placed on the glass, n = 1.52, how thick should the layer be to keep yellow-green light from being reflected?

7. A silicon solar cell has a nonreflective coating placed on it. If a film of sodium monoxide, n = 1.45, is placed on the silicon, n = 3.5, how thick should the layer be to keep yellow-green light (λ = 555 nm) from being reflected?

8. You can observe thin-film interference by dipping a bubble wand into some bubble solution and holding the wand in the air. What is the thickness of the thinnest soap film at which you would see a black stripe if the light illuminating the film has a wavelength of 521 nm? Use n = 1.33.

9. What is the thinnest soap film (n = 1.33) for which light of wavelength 521 nm will constructively interfere with itself?
10. **Film Thickness** Lucien is blowing bubbles and holds the bubble wand up so that a soap film is suspended vertically in the air. What is the second thinnest width of the soap film at which he could expect to see a bright stripe if the light illuminating the film has a wavelength of 575 nm? Assume the soap solution has an index of refraction of 1.33.

11. **Bright and Dark Patterns** Two very narrow slits are cut close to each other in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is seen on the paper. Describe how a wave behaves when it encounters a slit, and explain why some regions are bright while others are dark.

12. **Interference Patterns** Sketch the pattern described in problem 11.

13. **Interference Patterns** Sketch what happens to the pattern in problem 11 when the red light is replaced by blue light.

14. **Film Thickness** A plastic reflecting film \((n = 1.83)\) is placed on an auto glass window \((n = 1.52)\).

   a. What is the thinnest film that will reflect yellow-green light?

   b. Unfortunately, a film this thin cannot be manufactured. What is the next-thinnest film that will produce the same effect?

15. **Critical Thinking** The equation for wavelength from a double-slit experiment uses the simplification that \(\sin \theta \approx \tan \theta\). Up to what angle is this a good approximation when your data has two significant figures? Would the maximum angle for a valid approximation increase or decrease as you increase the precision of your angle measurement?
Objectives

- Explain how diffraction gratings form diffraction patterns.
- Describe how diffraction gratings are used in grating spectrometers.
- Discuss how diffraction limits the ability to distinguish two closely spaced objects with a lens.

Vocabulary

diffraction pattern

diffraction grating

In Chapter 16, you learned that smooth wave fronts of light spread when they are diffracted around an edge. Diffraction was explained using Huygens’ principle that a smooth wave front is made up of many small point-source wavelets. The cutting of coherent light on two edges spaced closely together produces a diffraction pattern, which is a pattern on a screen of constructive and destructive interference of Huygens’ wavelets.

**Single-Slit Diffraction**

When coherent, blue light passes through a single, small opening that is larger than the wavelength of the light, the light is diffracted by both edges, and a series of bright and dark bands appears on a distant screen, as shown in Figure 19-8. Instead of the nearly equally spaced bands produced by two coherent sources in Young’s double-slit experiment, this pattern has a wide, bright central band with dimmer, narrower bands on either side. When using red light instead of blue, the width of the bright central band increases. With white light, the pattern is a mixture of patterns of all the colors of the spectrum.

To see how Huygens’ wavelets produce the diffraction pattern, imagine a slit of width $w$ as being divided into an even number of Huygens’ points, as shown in Figure 19-9. Each Huygens’ point acts as a point source of Huygens’ wavelets. Divide the slit into two equal parts and choose one source from each part so that the pair is separated by a distance $w/2$. This pair of sources produces coherent, cylindrical waves that will interfere.

For any Huygens’ wavelet produced in the top half, there will be another Huygens’ wavelet in the bottom half, a distance $w/2$ away, that it will interfere destructively to create a dark band on the screen. All similar pairings of Huygens’ wavelets interfere destructively at dark bands. Conversely, a bright band on the screen is where pairings of Huygens’ wavelets interfere constructively. In the dim regions between bright and dark bands, partial destructive interference occurs.
Diffraction pattern When the single slit is illuminated, a central bright band appears at location $P_0$ on the screen, as shown in Figure 19-10. The first dark band is at position $P_1$. At this location, the path lengths $r_1$ and $r_2$ of the two Huygens’ wavelets differ by one-half wavelength, thereby producing a dark band by destructive interference. This model is mathematically similar to that of double-slit interference. A comparison of a single-slit diffraction pattern with a double-slit interference pattern using slits of the same width reveals that all of the bright interference fringes of the double-slit interference pattern fit within the width of the central bright band of the single-slit diffraction pattern. Double-slit interference results from the interference of the single-slit diffraction patterns of the two slits.

An equation now can be developed for the diffraction pattern produced by a single slit using the same simplifications that were used for double-slit interference, assuming that the distance to the screen is much larger than $w$. The separation distance between the sources of the two interfering waves is now $w/2$. To find the distance measured on the screen to the first dark band, $x_1$, note that the path length difference is now $\lambda/2$ because at the dark band there is destructive interference. As a result, $x_1/L = \lambda/w$.

You can see from Figure 19-10 that it might be difficult to measure the distance to the first dark band from the center of the central bright band. A better method of determining $x_1$ is to measure the width of the central bright band, $2x_1$. The following equation gives the width of the central bright band from single-slit diffraction.

**Width of Bright Band in Single-Slit Diffraction**

$$2x_1 = \frac{2\lambda L}{w}$$

The width of the central bright band is equal to the product of twice the wavelength times the distance to the screen, divided by the width of the slit.

Canceling the 2’s out of the above equation gives you the distance from the center of the central bright band to where the first dark band occurs. The location of additional dark bands can be found where the path lengths differ by $3\lambda/2$, $5\lambda/2$, and so on. This can be expressed as $x_m = m\lambda L/w$, where $m = 1, 2, 3$, etc., as limited by the small angle simplification. When $m = 1$, this equation provides the position of the first-order dark band. The second-order dark band occurs at $m = 2$, and so forth.
16. Monochromatic green light of wavelength 546 nm falls on a single slit with a width of 0.095 mm. The slit is located 75 cm from a screen. How wide will the central bright band be?

17. Yellow light with a wavelength of 589 nm passes through a slit of width 0.110 mm and makes a pattern on a screen. If the width of the central bright band is 2.60 \times 10^{-2} \text{ m}, how far is it from the slits to the screen?

18. Light from a He-Ne laser (\( \lambda = 632.8 \text{ nm} \)) falls on a slit of unknown width. A pattern is formed on a screen 1.15 m away, on which the central bright band is 15 mm wide. How wide is the slit?

19. Yellow light falls on a single slit 0.0295 mm wide. On a screen that is 60.0 cm away, the central bright band is 24.0 mm wide. What is the wavelength of the light?

20. White light falls on a single slit that is 0.050 mm wide. A screen is placed 1.00 m away. A student first puts a blue-violet filter (\( \lambda = 441 \text{ nm} \)) over the slit, then a red filter (\( \lambda = 622 \text{ nm} \)). The student measures the width of the central bright band.
   a. Which filter produced the wider band?
   b. Calculate the width of the central bright band for each of the two filters.

Single-slit diffraction patterns make the wave nature of light noticeable when the slits are 10 to 100 times the wavelength of the light. Larger openings, however, cast sharp shadows, as Isaac Newton first observed. While the single-slit pattern depends on the wavelength of light, it is only when a large number of slits are put together that diffraction provides a useful tool for measuring wavelength.
Diffraction Gratings

Although double-slit interference and single-slit diffraction depend on the wavelength of light, diffraction gratings, such as those shown in Figure 19-11, are used to make precision measurements of wavelength. A **diffraction grating** is a device made up of many single slits that diffract light and form a diffraction pattern that is an overlap of single-slit diffraction patterns. Diffraction gratings can have as many as 10,000 slits per centimeter. That is, the spacing between the slits can be as small as $10^{-6}$ m, or 1000 nm.

One type of diffraction grating is called a transmission grating. A transmission grating can be made by scratching very fine lines with a diamond point on glass that transmits light. The spaces between the scratched lines act like slits. A less expensive type of grating is a replica grating. A replica grating is made by pressing a thin plastic sheet onto a glass grating. When the plastic is pulled away, it contains an accurate imprint of the scratches. Jewelry made from replica gratings, shown in Figure 19-12a, produces a spectrum.

Reflection gratings are made by inscribing fine lines on metallic or reflective glass surfaces. The color spectra produced when white light reflects off the surface of a CD or DVD is the result of a reflection grating, as shown in Figure 19-12b. If you were to shine monochromatic light on a DVD, the reflected light would produce a diffraction pattern on a screen. Transmission and reflection gratings produce similar diffraction patterns, which can be analyzed in the same manner.

Holographic diffraction gratings produce the brightest spectra. They are made by using a laser and mirrors to create an interference pattern consisting of parallel bright and dark lines. The pattern is projected on a piece of metal that is coated with a light-sensitive material. The light produces a chemical reaction that hardens the material. The metal is then placed in acid, which attacks the metal wherever it was not protected by the hardened material. The result is a series of hills and valleys in the metal identical to the original interference pattern. The metal can be used as a reflection grating or a replica transmission grating can be made from it. Because of the sinusoidal shape of the hills and valleys, the diffraction patterns are exceptionally bright.

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**Figure 19-11** Diffraction gratings are used to create diffraction patterns for the analysis of light sources.

**Figure 19-12** A transmission grating spectrum is created by jewelry made with replica gratings (a). Compact discs act as reflection gratings, creating a spectrum diffraction pattern when they are placed under white light (b).
A spectroscope is used to measure the wavelengths of light emitted by a light source.

An instrument used to measure light wavelengths using a diffraction grating is called a grating spectroscope, as shown in the diagram in Figure 19-13. The source to be analyzed emits light that is directed to a slit. The light from the slit passes through a diffraction grating. The grating produces a diffraction pattern that is viewed through a telescope.

The diffraction pattern produced by a diffraction grating has narrow, equally spaced, bright lines, as shown in Figure 19-14. The larger the number of slits per unit length of the grating, the narrower the lines in the diffraction pattern. As a result, the distance between the bright lines can be measured much more precisely with a grating spectroscope than with a double slit.

Earlier in this chapter, you found that the interference pattern produced by a double slit could be used to calculate wavelength. An equation for the diffraction grating can be developed in the same way as for the double slit. However, with a diffraction grating, $\theta$ could be large, so the small angle simplification does not apply. Wavelength can be found by measuring the angle, $\theta$, between the central bright line and the first-order bright line.

The wavelength of light is equal to the slit separation distance times the sine of the angle at which the first-order bright line occurs.

Constructive interference from a diffraction grating occurs at angles on either side of the central bright line given by the equation $m\lambda = d\sin \theta$, where $m = 0, 1, 2, \text{ etc.}$ The central bright line occurs at $m = 0$. 

Figure 19-14 A grating was used to produce diffraction patterns for red light (a) and white light (b).
21. White light shines through a grating onto a screen. Describe the pattern that is produced.

22. If blue light of wavelength 434 nm shines on a diffraction grating and the spacing of the resulting lines on a screen that is 1.05 m away is 0.55 m, what is the spacing between the slits in the grating?

23. A diffraction grating with slits separated by $8.60 \times 10^{-7} \text{ m}$ is illuminated by violet light with a wavelength of 421 nm. If the screen is 80.0 cm from the grating, what is the separation of the lines in the diffraction pattern?

24. Blue light shines on the DVD in Example Problem 3. If the dots produced on a wall that is 0.65 m away are separated by 58.0 cm, what is the wavelength of the light?

25. Light of wavelength 632 nm passes through a diffraction grating and creates a pattern on a screen that is 0.55 m away. If the first bright band is 5.6 cm from the central bright band, how many slits per centimeter does the grating have?
In thin-film interference, the interference pattern is visible only within a narrow angle of view straight over the film. This would be the case for the *Morpho* butterfly’s blue, shimmering interference pattern, if not for the layer of glass-like scales on top of the layer of ground scales. This glass-like scale layer acts as a diffraction grating and causes the blue, shimmering interference pattern to be spread to a diffraction pattern with a wider angle of view. Scientists believe that this makes the *Morpho* butterfly more visible to potential mates.

### Resolving Power of Lenses

The circular lens of a telescope, a microscope, and even your eye acts as a hole, called an aperture, through which light is allowed to pass. An aperture diffracts the light, just as a single slit does. Alternating bright and dark rings occur with a circular aperture, as shown in Figure 19-15. The equation for an aperture is similar to that for a single slit. However, an aperture has a circular edge rather than the two edges of a slit, so slit width, \( w \), is replaced by aperture diameter, \( D \), and an extra geometric factor of 1.22 enters the equation, resulting in

\[
x_1 = \frac{1.22 \lambda L}{D}
\]

When light from a distant star is viewed through the aperture of a telescope, the image is spread out due to diffraction. If two stars are close enough together, the images may be blurred together, as shown in Figure 19-16. In 1879, Lord Rayleigh, a British physicist, mathematician, and Nobel prize winner, established a criterion for determining whether there is one or two stars in such an image. The **Rayleigh criterion** states that if the center of the bright spot of one star’s image falls on the first dark ring of the second, the two images are at the limit of resolution. That is, a viewer will be able to tell that there are two stars rather than only one.

If two images are at the limit of resolution, how far apart are the objects? Using the Rayleigh criterion, the centers of the bright spots of the two images are a distance of \( x_1 \) apart. Figure 19-16 shows that similar triangles can be used to find that

\[
x_{\text{obj}}/L_{\text{obj}} = x_1/L
\]

Combining this with \( x_1 = \frac{1.22 \lambda L}{D} \) to eliminate \( x_1/L \), and solving for the separation distance between objects, \( x_{\text{obj}} \), the following equation can be derived.

**Rayleigh Criterion**

\[
x_{\text{obj}} = \frac{1.22 \lambda L_{\text{obj}}}{D}
\]

The separation distance between objects that are at the limit of resolution is equal to 1.22, times the wavelength of light, times the distance from the circular aperture to the objects, divided by the diameter of the circular aperture.
**Diffraction in the eye** In bright light the eye’s pupil is about 3 mm in diameter. The eye is most sensitive to yellow-green light where \( \lambda = 550 \text{ nm} \). So the Rayleigh criterion applied to the eye gives \( x_{\text{obj}} = 2 \times 10^{-4} I_{\text{obj}} \). The distance between the pupil and retina is about 2 cm, so two barely resolved point sources would be separated by about \( 4 \mu\text{m} \) on the retina. The spacing between the light detectors, the cones, in the most sensitive part of the eye, the fovea, is about \( 2 \mu\text{m} \). Thus, in the ideal case, the three adjacent cones would record light, dark, and light. It would seem that the eye is ideally constructed. If cones were closer together, they would see details of the diffraction pattern, not of the sources. If cones were farther apart, they would not be able to resolve all possible detail.

Applying the Rayleigh criterion to find the ability of the eye to separate two distance sources shows that the eye could separate two automobile headlamps (1.5 m apart) at a distance of 7 km. In practice, however, the eye is not limited by diffraction. Imperfections in the lens and the liquid that fills the eye reduce the eye’s resolution to about five times that set by the Rayleigh criterion. Most people use their eyes for purposes other than resolving point sources. For example, the eye seems to have a built-in ability to detect straight edges.

Many telescope manufacturers advertise that their instruments are diffraction limited. That is, they claim that their telescopes can separate two point sources at the Rayleigh criterion. To reach this limit they must grind the mirrors and lenses to an accuracy of one-tenth of a wavelength, or about 55 nm. The larger the diameter of the mirror, the greater the resolution of the telescope. Unfortunately, light from planets or stars must go through Earth’s atmosphere. The same variations in the atmosphere that cause stars to twinkle keep telescopes from reaching the diffraction limit. Because the *Hubble Space Telescope* is above Earth’s atmosphere, the resolution of its images is much better than those of larger telescopes on Earth’s surface.

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**19.2 Section Review**

26. **Distance Between First-Order Dark Bands**

Monochromatic green light of wavelength 546 nm falls on a single slit of width 0.080 mm. The slit is located 68.0 cm from a screen. What is the separation of the first dark bands on each side of the central bright band?

27. **Diffraction Patterns**

Many narrow slits are close to each other and equally spaced in a large piece of cardboard. They are illuminated by monochromatic red light. A sheet of white paper is placed far from the slits, and a pattern of bright and dark bands is visible on the paper. Sketch the pattern that would be seen on the screen.

28. **Line Spacing**

You shine a red laser light through one diffraction grating and form a pattern of red dots on a screen. Then you substitute a second diffraction grating for the first one, forming a different pattern. The dots produced by the first grating are spread out more than those produced by the second. Which grating has more lines per millimeter?

29. **Rayleigh Criterion**

The brightest star in the winter sky in the northern hemisphere is Sirius. In reality, Sirius is a system of two stars that orbit each other. If the *Hubble Space Telescope* (diameter 2.4 m) is pointed at the Sirius system, which is 8.44 light-years from Earth, what is the minimum separation there would need to be between the stars in order for the telescope to be able to resolve them? Assume that the average light coming from the stars has a wavelength of 550 nm.

30. **Critical Thinking**

You are shown a spectrometer, but do not know whether it produces its spectrum with a prism or a grating. By looking at a white-light spectrum, how could you tell?
Double-Slit Interference of Light

Light sometimes behaves as a wave. As coherent light strikes a pair of slits that are close together, the light passing through the slits will create a pattern of constructive and destructive interference on a screen. In this investigation you will develop a procedure and measure the wavelength of a monochromatic light source using two slits.

QUESTION
How can a double-slit interference pattern of light be used to measure the light’s wavelength?

Objectives
- **Observe** a double-slit interference pattern of monochromatic light.
- **Calculate** the wavelength of light using a double-slit interference pattern.

Safety Precautions
- Use laser protective eyewear approved by ANSI.
- Never look directly into the light of a laser.

Possible Materials
- Laser pointer or laser to be tested
- Double-slit plate
- Laser pointer or laser of known wavelength
- Clothes pin to hold a laser pointer
- Clay ball to hold the double-slit plate
- Meterstick

Procedure
1. Determine which equation applies to double-slit interference.
2. Use a double slit of known slit-separation distance, \( d \), or develop a method to determine \( d \).
3. Sketch how light passes through a double slit to help you determine how \( x \) and \( L \) can be measured.
4. Using your sketch from step 3 and the list of possible materials provided in this lab, design the lab setup and write a procedure for performing the experiment.
5. Determine the values of \( m \) that would be invalid for the equation.
6. **CAUTION:** Looking directly into laser light could damage your eyes.
7. Be sure to check with your teacher and have approval before you implement your design.
8. Perform your experiment. Write your data in a data table similar to the one on the next page.
1. Adjust the distance of your slits from the screen. Is there a distance that allows you to collect the most data with the best precision?

2. Calculate the wavelength, \( \lambda \), of your light source using \( m \) and measurements of \( x \), \( d \), and \( L \).

3. Error Analysis Compare your calculated wavelength to the accepted value by determining the percentage of error.

Conclude and Apply

1. Conclude Did your procedure enable you to use a double-slit interference pattern to measure the wavelength of light? Explain.

2. Estimate what results you would get if you used a plate with a smaller slit separation distance, \( d \), and performed the experiment exactly the same.

3. Infer How would your observations change if you used green light, but used the same double-slit plate and screen distance? What would you observe?

Going Further

1. Use a Scientific Explanation Describe why the double-slit interference pattern dims, brightens, and dims again as distance from the center of the pattern increases.

2. Error Analysis Describe several things you could do in the future to reduce systematic error in your experiment.

3. Evaluate Examine the measuring equipment you used and determine which equipment limited you the most on the precision of your calculations and which equipment gave you more precision than you needed, if any.

4. Lab Techniques What might be done to an experimental setup to use white light from a normal lightbulb to produce a double-slit interference pattern?

Real-World Physics

1. When white light shines through slits in a screen door, why is a pattern not visible in the shadow on a wall?

2. Would things look different if all of the light that illuminated your world was coherent? Explain.

Physics Online

To find out more about interference patterns, visit the Web site: physicspp.com
Holography

Holography is a form of photography that produces a three-dimensional image. Dennis Gabor made the first hologram in 1947, but holography was impractical until the invention of the gas laser in 1960. Holograms are used on credit cards to help prevent counterfeiting, and they may one day be used for ultra-high-density data storage. How is a hologram made?

1. Laser light strikes a semitransparent mirror known as a beam splitter. This creates two coherent beams.
2. The reference and object beams are directed by mirrors, and are made to diverge by lenses.
3. Light scattered by reflection off the object, in this case a basket of pears, interferes with the reference beam. The interference pattern that is formed by the two beams is recorded on the holographic exposure plate.
4. When a transparent film of the developed plate is placed in a diverging laser beam, light passing through the film creates a three-dimensional virtual image of the original object with rainbowlike bands of color.
5. A person sees the image as if viewing the original object through a window. Moving his or her head changes the perspective.

Thinking Critically

1. Infer A hologram records a complex pattern of constructive and destructive interference fringes. Why do you suppose a vibration-isolated surface is needed for good results?
2. Use Scientific Explanations Identify and explain where the following wave properties occur in the diagrams: reflection, refraction, and interference.
### 19.1 Interference

**Vocabulary**
- incoherent light (p. 515)
- coherent light (p. 516)
- interference fringes (p. 516)
- monochromatic light (p. 516)
- thin-film interference (p. 520)

**Key Concepts**
- Incoherent light illuminates an object evenly, just as a lightbulb illuminates your desk.
- Only the superposition of light waves from coherent light sources can produce an interference pattern.
- Interference demonstrates that light has wave properties.
- Light passing through two closely spaced, narrow slits produces a pattern on a screen of dark and light bands called interference fringes.
- Interference patterns can be used to measure the wavelength of light.

\[
\lambda = \frac{2d}{L}
\]

- Interference patterns can result from the creation of coherent light at the refractive boundary of a thin film.

### 19.2 Diffraction

**Vocabulary**
- diffraction pattern (p. 524)
- diffraction grating (p. 527)
- Rayleigh criterion (p. 530)

**Key Concepts**
- Light passing through a narrow slit is diffracted, or spread out from a straight-line path, and produces a diffraction pattern on a screen.
- The diffraction pattern from a single slit has a bright central band that has a width equal to the distance between the first dark bands on either side of the bright central band.

\[
2x_1 = \frac{2\lambda L}{w}
\]

- Diffraction gratings consist of large numbers of slits that are very close together and produce narrow lines that result from an overlap of the single-slit diffraction patterns of all of the slits in the grating.
- Diffraction gratings can be used to measure the wavelength of light precisely or to separate light composed of different wavelengths.

\[
\lambda = d \sin \theta
\]

- Diffraction limits the ability of an aperture to distinguish two closely spaced objects.

\[
x_{\text{obj}} = \frac{1.22\lambda L_{\text{obj}}}{D}
\]

- If the central bright spot of one image falls on the first dark ring of the second image, the images are at the limit of resolution.
Concept Mapping

31. Monochromatic light of wavelength $\lambda$ illuminates two slits in a Young's double-slit experiment setup that are separated by a distance, $d$. A pattern is projected onto a screen a distance, $L$, away from the slits. Complete the following concept map using $\lambda$, $L$, and $d$ to indicate how you could vary them to produce the indicated change in the spacing between adjacent bright bands, $x$.

![Concept Map Diagram]

Applying Concepts

40. For each of the following examples, indicate whether the color is produced by thin-film interference, refraction, or the presence of pigments.
   a. soap bubbles
   b. rose petals
   c. oil films
   d. a rainbow

41. How can you tell whether a pattern is produced by a single slit or a double slit?

42. Describe the changes in a single-slit diffraction pattern as the width of the slit is decreased.

43. Science Fair At a science fair, one exhibition is a very large soap film that has a fairly consistent width. It is illuminated by a light with a wavelength of 432 nm, and nearly the entire surface appears to be a lovely shade of purple. What would you see in the following situations?
   a. the film thickness was doubled
   b. the film thickness was increased by half a wavelength of the illuminating light
   c. the film thickness was decreased by one quarter of a wavelength of the illuminating light

44. What are the differences in the characteristics of the diffraction patterns formed by diffraction gratings containing $10^4$ lines/cm and $10^5$ lines/cm?

45. Laser Pointer Challenge You have two laser pointers, a red one and a green one. Your friends Mark and Carlos disagree about which has the longer wavelength. Mark insists that red light has a longer wavelength, while Carlos is sure that green has the longer wavelength. You have a diffraction grating handy. Describe what demonstration you would do with this equipment and how you would explain the results to Carlos and Mark to settle their disagreement.

46. Optical Microscope Why is blue light used for illumination in an optical microscope?

Mastering Concepts

32. Why is it important that monochromatic light was used to make the interference pattern in Young's interference experiment? (19.1)

33. Explain why the position of the central bright band of a double-slit interference pattern cannot be used to determine the wavelength of the light waves. (19.1)

34. Describe how you could use light of a known wavelength to find the distance between two slits. (19.1)

35. Describe in your own words what happens in thin-film interference when a dark band is produced by light shining on a soap film suspended in air. Make sure you include in your explanation how the wavelength of the light and thickness of the film are related. (19.1)

36. White light shines through a diffraction grating. Are the resulting red lines spaced more closely or farther apart than the resulting violet lines? Why? (19.2)

37. Why do diffraction gratings have large numbers of slits? Why are these slits so close together? (19.2)

38. Why would a telescope with a small diameter not be able to resolve the images of two closely spaced stars? (19.2)

39. For a given diffraction grating, which color of visible light produces a bright line closest to the central bright band? (19.2)

Mastering Problems

19.1 Interference

47. Light falls on a pair of slits 19.0 $\mu$m apart and 80.0 cm from a screen, as shown in Figure 19-17. The first-order bright band is 1.90 cm from the central bright band. What is the wavelength of the light?
48. **Oil Slick** After a short spring shower, Tom and Ann take their dog for a walk and notice a thin film of oil ($n = 1.45$) on a puddle of water, producing different colors. What is the minimum thickness of a place where the oil creates constructive interference for light with a wavelength of 545 nm?

49. Light of wavelength 542 nm falls on a double slit. First-order bright bands appear 4.00 cm from the central bright band. The screen is 1.20 m from the slits. How far apart are the slits?

50. **Insulation Film** Winter is approaching and Alejandro is helping to cover the windows in his home with thin sheets of clear plastic ($n = 1.81$) to keep the drafts out. After the plastic is taped up around the windows such that there is air between the plastic and the glass panes, the plastic is heated with a hair dryer to shrink-wrap the window. The thickness of the plastic is altered during this process. Alejandro notices a place on the plastic where there is a blue stripe of color. He realizes that this is created by thin-film interference. What are three possible thicknesses of the portion of the plastic where the blue stripe is produced if the wavelength of the light is 4.40 $\times$ 10$^{-2}$ nm?

51. Samir shines a red laser pointer through three different double-slit setups. In setup A, the slits are separated by 0.150 mm and the screen is 0.60 m away from the slits. In setup B, the slits are separated by 0.175 mm and the screen is 0.80 m away. Setup C has the slits separated by 0.150 mm and the screen a distance of 0.80 m away. Rank the three setups according to the separation between the central bright band and the first-order bright band, from least to most separation. Specifically indicate any ties.

### 19.2 Diffraction

52. Monochromatic light passes through a single slit with a width of 0.010 cm and falls on a screen 100 cm away, as shown in Figure 19-18. If the width of the central band is 1.20 cm, what is the wavelength of the light?

53. A good diffraction grating has 2.5 $\times$ 10$^3$ lines per cm. What is the distance between two lines in the grating?

54. Light with a wavelength of 4.5 $\times$ 10$^{-5}$ cm passes through a single slit and falls on a screen 100 cm away. If the slit is 0.015 cm wide, what is the distance from the center of the pattern to the first dark band?

55. **Hubble Space Telescope** Suppose the Hubble Space Telescope, 2.4 m in diameter, is in orbit 1.0 $\times$ 10$^5$ m above Earth and is turned to view Earth, as shown in Figure 19-19. If you ignore the effect of the atmosphere, how large an object can the telescope resolve? Use $\lambda = 5.1 \times 10^{-7}$ m.

[Figure 19-19]

56. Monochromatic light with a wavelength of 425 nm passes through a single slit and falls on a screen 75 cm away. If the central bright band is 0.60 cm wide, what is the width of the slit?

57. **Kaleidoscope** Jennifer is playing with a kaleidoscope from which the mirrors have been removed. The eyehole at the end is 7.0 mm in diameter. If she can just distinguish two bluish-purple specks on the other end of the kaleidoscope separated by 40 $\mu$m, what is the length of the kaleidoscope? Use $\lambda = 650$ nm and assume that the resolution is diffraction limited through the eyehole.

58. **Spectroscope** A spectroscope uses a grating with 12,000 lines/cm. Find the angles at which red light, 632 nm, and blue light, 421 nm, have first-order bright lines.
Mixed Review

59. **Record** Marie uses an old $33\frac{1}{3}$ rpm record as a diffraction grating. She shines a laser, $\lambda = 632.8$ nm, on the record, as shown in Figure 19-20. On a screen 4.0 m from the record, a series of red dots 21 mm apart are visible.
   a. How many ridges are there in a centimeter along the radius of the record?
   b. Marie checks her results by noting that the ridges represent a song that lasts 4.01 minutes and takes up 16 mm on the record. How many ridges should there be in a centimeter?

60. An anti-reflective coating, $n = 1.2$, is applied to a lens. If the thickness of the coating is 125 nm, what is (are) the color(s) of light for which complete destructive interference will occur? *Hint: Assume the lens is made out of glass.*

61. **Camera** When a camera with a 50-mm lens is set at f/8, its aperture has an opening 6.25 mm in diameter.
   a. For light with $\lambda = 550$ nm, what is the resolution of the lens? The film is 50.0 mm from the lens.
   b. The owner of a camera needs to decide which film to buy for it. The expensive one, called fine-grained film, has 200 grains/mm. The less costly, coarse-grained film has only 50 grains/mm. If the owner wants a grain to be no smaller than the width of the central bright spot calculated in part a, which film should he purchase?

Thinking Critically

62. **Apply Concepts** Yellow light falls on a diffraction grating. On a screen behind the grating, you see three spots: one at zero degrees, where there is no diffraction, and one each at $+30^\circ$ and $-30^\circ$. You now add a blue light of equal intensity that is in the same direction as the yellow light. What pattern of spots will you now see on the screen?

63. **Apply Concepts** Blue light of wavelength $\lambda$ passes through a single slit of width $w$. A diffraction pattern appears on a screen. If you now replace the blue light with a green light of wavelength $1.5\lambda$, to what width should you change the slit to get the original pattern back?

64. **Analyze and Conclude** At night, the pupil of a human eye has an aperture diameter of 8.0 mm. The diameter is smaller in daylight. An automobile’s headlights are separated by 1.8 m.
   a. Based upon Rayleigh’s criterion, how far away can the human eye distinguish the two headlights at night? *Hint: Assume a wavelength of 525 nm.*
   b. Can you actually see a car’s headlights at the distance calculated in part a? Does diffraction limit your eyes’ sensing ability? Hypothesize as to what might be the limiting factors.

Writing in Physics

65. Research and describe Thomas Young’s contributions to physics. Evaluate the impact of his research on the scientific thought about the nature of light.

66. Research and interpret the role of diffraction in medicine and astronomy. Describe at least two applications in each field.

Cumulative Review

67. How much work must be done to push a 0.5-m$^3$ block of wood to the bottom of a 4-m-deep swimming pool? The density of wood is 500 kg/m$^3$. (Chapter 13)

68. What are the wavelengths of microwaves in an oven if their frequency is 2.4 GHz? (Chapter 14)

69. Sound wave crests that are emitted by an airplane are 1.00 m apart in front of the plane, and 2.00 m apart behind the plane. (Chapter 15)
   a. What is the wavelength of the sound in still air?
   b. If the speed of sound is 330 m/s, what is the frequency of the source?
   c. What is the speed of the airplane?

70. A concave mirror has a 48.0-cm radius. A 2.0-cm-tall object is placed 12.0 cm from the mirror. Calculate the image position and image height. (Chapter 17)

71. The focal length of a convex lens is 21.0 cm. A 2.00-cm-tall candle is located 7.50 cm from the lens. Use the thin-lens equation to calculate the image position and image height. (Chapter 18)
**Multiple Choice**

1. What is the best possible explanation for why the colors of a thin film, such as a soap bubble or oil on water, appear to change and move as you watch?
   - because convective heat waves in the air next to the thin film distort the light
   - because the film thickness at any given location changes over time
   - because the wavelengths in sunlight vary over time
   - because your vision varies slightly over time

2. Light at 410 nm shines through a slit and falls on a flat screen as shown in the figure below. The width of the slit is $3.8 \times 10^{-6}$ m. What is the width of the central bright band?
   - 0.024 m
   - 0.048 m
   - 0.031 m
   - 0.063 m

3. For the situation in problem 2, what is the angle, $\theta$, of the first dark band?
   - 3.1°
   - 12.4°
   - 6.2°
   - 17°

4. Two stars $6.2 \times 10^4$ light-years from Earth are 3.1 light-years apart. What is the smallest diameter telescope that could resolve them using 610 nm light?
   - $5.0 \times 10^{-5}$ m
   - $1.5 \times 10^{-2}$ m
   - $6.1 \times 10^{-5}$ m
   - $1.5 \times 10^7$ m

5. A grating has slits that are 0.055 mm apart. What is the angle of the first-order bright line for light with a wavelength of 650 nm?
   - 0.012°
   - 1.0°
   - 0.68°
   - 11°

6. A laser beam at 638 nm illuminates two narrow slits. The third-order band of the resulting pattern is 7.5 cm from the center bright band. The screen is 2.475 m from the slits. How far apart are the slits?
   - $5.8 \times 10^{-8}$ m
   - $2.1 \times 10^{-5}$ m
   - $6.3 \times 10^{-7}$ m
   - $6.3 \times 10^{-5}$ m

7. A flat screen is placed 4.200 m from a pair of slits that are illuminated by a beam of monochromatic light. On the screen, the separation between the central bright band and the second-order bright band is 0.082 m. The distance between the slits is $5.3 \times 10^{-3}$ m. Determine the wavelength of the light.
   - $2.6 \times 10^{-7}$ m
   - $6.2 \times 10^{-7}$ m
   - $5.2 \times 10^{-7}$ m
   - $1.0 \times 10^{-6}$ m

8. A clown is blowing soap bubbles and you notice that the color of one region of a particularly large bubble matches the color of his nose. If the bubble is reflecting $6.5 \times 10^{-7}$ m red light waves, and the index of refraction of the soap film is 1.41, what is the minimum thickness of the soap bubble at the location where it is reflecting red?
   - $1.2 \times 10^{-7}$ m
   - $9.2 \times 10^{-7}$ m
   - $3.5 \times 10^{-7}$ m
   - $1.9 \times 10^{-6}$ m

9. A diffraction grating that has 6000 slits per cm produces a diffraction pattern that has a first-order bright line at 20° from the central bright line. What is the wavelength of the light?