CLT for Means:

1. If SRS of size \( n \), where \( n \geq 30 \), are drawn from any population with mean \( \mu \) and a standard deviation \( \sigma \), then the sampling distribution of sample means approximates a normal distribution. The greater the sample size, the better the approximation.

2. If the population itself is normally distributed, the sampling distribution of sample means is normally distributed for any sample size \( n \).

In either case, the sampling distribution of sample means is approximately normal and has a mean equal to the population mean:
\[
\mu_x = \mu
\]

...and a standard deviation equal to the population standard deviation divided by the square root of \( n \):
\[
\sigma_x = \frac{\sigma}{\sqrt{n}}
\]

1.) A manufacturer claims that their batteries last an average of 200 hours. 34 batteries are chosen at random and are tested. The average battery life of the sample was 198 hours with a standard deviation of 2.4 hours.
   a. Determine (and label) the statistic(s) and parameter(s) in the sampling distribution described above.
   b. Determine whether or not the CLT applies. If yes, find \( \mu_x \) and \( \sigma_x \); if no, explain why.
   c. Explain how a Confidence Interval could be used to test the manufacturer’s claim.

2.) Suppose you administer a certain aptitude test to a simple random sample of 9 students in your school, and that the average score is 105. From past experience, scores on such a test among students like those at your school follow a Normal distribution with \( \sigma = 15 \). We want to determine the mean score \( \mu \) of the population. Does the CLT apply? Why or why not?

3.) To cut the standard deviation of \( \bar{x} \) in half, you must take a sample _____ times as large.

4.) What does the central limit theorem say about the shape of the sampling distribution of \( \bar{x} \)?
Steps and Conditions for constructing a Confidence Interval for $\mu$:

1. Identify $n$
2. Identify $\bar{x}$
3. Verify the sampling distribution of $\bar{x}$ can be approximated by the normal distribution. In other words, verify that either: $n \geq 30$ or the population is normally distributed.
4. Find the Critical Value, $z^*$, which corresponds to the given level of confidence:
   
<table>
<thead>
<tr>
<th>Level of confidence</th>
<th>Critical Value, $z^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>1.645</td>
</tr>
<tr>
<td>95%</td>
<td>1.96</td>
</tr>
<tr>
<td>99%</td>
<td>2.575</td>
</tr>
</tbody>
</table>

5. Determine the Error Estimate, $E$, or how far to the left and right of the sample statistic we will go to estimate the population parameter. This is determined by multiplying the number of standard deviations ($z^*$) by the amount of a standard deviation, $\sigma_x$:
   
   $$E = z^* \frac{\sigma}{\sqrt{n}}$$

   OR
   $$E = z^* \frac{s}{\sqrt{n}}$$

   if $\sigma$ is known and $n \geq 30$

   OR
   $$E = z^* \frac{s}{\sqrt{n}}$$

   if $\sigma$ is known for any $n$ IF the population is approximately normally distributed.

   OR
   $$E = z^* \frac{s}{\sqrt{n}}$$

   if $\sigma$ is Unknown and $n \geq 30$

6. Find the left and right endpoints and form the confidence interval:

   - Left endpoint: $\bar{x} - E$
   - Right endpoint: $\bar{x} + E$
   - Confidence Interval: $\bar{x} \pm E$

7. Write the Confidence Interval in sentence format:
   
   * I am ___% confident that the true mean of (insert characteristic of interest) is between ($\bar{x} - E$ and $\bar{x} + E$).
   OR
   * I am ___% confident that ($\bar{x} - E < \mu < \bar{x} + E$).

NOTE: There is no way to know whether the confidence interval you constructed actually “catches” the true mean, but we can say that the method will succeed in capturing the unknown population parameter about ___% of the time.

5.) A study of the career paths of hotel general managers sent questionnaires to an SRS of hotels belonging to major U.S. hotel chains. There were 114 responses. The average time these 114 general managers had spent with their current company was 11.78 years. Construct and interpret a 99% confidence interval for the mean number of years general managers of major-chain hotels have spent with their current company. ($\sigma$ is known to be 3.2 years).

6.) High school students who take the SAT Mathematics exam a second time generally score higher than on their first try. The change in score has a Normal distribution with standard deviation $\sigma = 50$. A random sample of 1000 students gain an average of $\bar{x} = 22$ points on their second try. Construct and interpret a 95% confidence interval for the mean score gain, $\mu$, in the population.

7.) Refer to #6 above. Suppose the same result, $\bar{x} = 22$, had come from a sample of 250 students. Calculate the 95% confidence interval for the population mean $\mu$ in this case.

8.) Refer to #6 above. Suppose the same result, $\bar{x} = 22$, had come from a sample of 4000 students. Again, calculate the 95% confidence interval for the population mean $\mu$ in this case.

9.) What are the margins of error for samples of size 250, 1000, and 4,000? How does increasing the sample size affect the margin of error of a confidence interval?

10.) Suppose you administer a certain aptitude test to a simple random sample of 9 students in your school, and that the average score is 105. From past experience, scores on such a test among students like those at your school follow a Normal distribution with $\sigma = 15$. We want to determine the mean score $\mu$ of the population. What sample size would be needed to have a margin of error of at most 4 points (with 95% confidence)?