A compound inequality is an inequality that is formed by the union, “or,” or the intersection, “and,” of two simple inequalities.

**Unions**
When an inequality is combined by the word “or” the compound inequality is formed. This union is worked out as two separate inequalities and then graphed on a common number line.

**Ex1)** \(4x + 3 < 11 \text{ or } 3x - 4 > 8\)  
\[
\begin{align*}
4x + 3 &< 11 \\
4x &< 8 \\
x &< 2
\end{align*}
\]

\[
\begin{align*}
3x - 4 &> 8 \\
3x &> 12 \\
x &> 4
\end{align*}
\]

\[
\begin{align*}
x < 2 \text{ OR } x > 4
\end{align*}
\]

**Ex2)** \(3x + 1 < 4 \text{ or } 2x - 5 > 7\)  
\[
\begin{align*}
3x + 1 &< 4 \\
3x &< 3 \\
x &< 1
\end{align*}
\]

\[
\begin{align*}
2x - 5 &> 7 \\
x &> 12 \\
x &< 6
\end{align*}
\]

**Intersections**
When an inequality is combined by the word “and” the compound inequality is formed. However, usually the inequality is written in compact form indicating it is a compound inequality.

**Ex3)** \(-2 \leq 3x - 8 \leq 10\)  
\[
\begin{align*}
-2 &\leq 3x - 8 \\
8 &\geq 3x \\
2 &\geq x
\end{align*}
\]

**Ex4)** \(8 \leq 2x + 6 \leq 18\)  
\[
\begin{align*}
8 &\leq 2x + 6 \\
-6 &\geq 2x \\
-6 &\geq x
\end{align*}
\]

**Writing in Compact Form**
There are times when the function does have the word and between the inequalities and you have to write them in compact form. Generally when writing an inequality, the least number is on the left and the greater number is on the right. Below are the solutions that you need to write in compact form.

**Ex5)** \(x > 3 \text{ and } x < 6\)  
\[
3 < x < 6
\]

**Ex6)** \(x \geq -2 \text{ and } x \leq 1\)  
\[
-2 \leq x \leq 1
\]

**Ex7)** \(x \leq 3 \text{ and } x \geq 0\)  
\[
0 \leq x \leq 3
\]

**Ex8)** \(x \geq -6 \text{ and } x < -2\)  
\[
-6 \leq x < -2
\]

**Ex9)** \(x \leq 4 \text{ and } x > 2\)  
\[
2 < x \leq 4
\]

**Ex10)** \(x > -4 \text{ and } x \leq 0\)  
\[
-4 < x \leq 0
\]
Compound Inequalities in Context

Ex11) Luke and Logan play on the same baseball team. They practice at the Lyons Park ball field. Luke lives 3 miles from the field and Logan lives 2 miles from the field.

a) Plot a point to represent the location of the Lyons Park ball field.

b) Use your point that represents Lyons Park and draw a circle to represent all the possible places Luke could live.

c) Use your point that represents Lyons Park, and draw another circle to represent all the possible places Logan could live.

-2 miles

-3 miles

Possible location of Luke’s house

Possible location of Logan’s house

3 miles

d) What is the shortest distance, d, that could separate their homes? **1 mile**

e) What is the longest distance, d, that could separate their homes? **5 miles**

f) Write a compound inequality that would represent all the possible distances that could separate their homes.

\[ 1 \leq d \leq 5 \]

Practice:

1) \(-3 \leq \frac{x}{2} + 1 < 2\)

\[-4 \leq \frac{x}{2} < 1\]

\[-8 \leq x < 2\]

2) \(7x - 5 \geq 65\) or \(-3x - 2 \geq -2\)

\(7x \geq 70\)

\(-3x \geq 0\)

\(x \geq 10\) or \(x \leq 0\)

3) \(-1 + 5x > -26\) and \(7x - 2 \leq 12\)

\(5x > -25\)

\(7x \leq 14\)

\(x > -5\) and \(x \leq 2\)

\(-5 \leq x \leq 2\)

4) \(-1 - 10x < -1\) or \(10 + 3x \leq -5\)

\(-10x < 0\)

\(3x \leq -15\)

\(x > 0\) or \(x \leq -5\)