Using and Expressing Measurements

Your height (67 inches), your weight (134 pounds), and the speed you drive on the highway (65 miles/hour) are some familiar examples of measurements. A measurement is a quantity that has both a number and a unit. Everyone makes and uses measurements. For instance, you decide how to dress in the morning based on the temperature outside. If you were baking cookies, you would measure the volumes of the ingredients as indicated in the recipe.

Such everyday situations are similar to those faced by scientists. Measurements are fundamental to the experimental sciences. For that reason, it is important to be able to make measurements and to decide whether a measurement is correct. The units typically used in the sciences are those of the International System of Units (SI).

In chemistry, you will often encounter very large or very small numbers. A single gram of hydrogen, for example, contains approximately 602,000,000,000,000,000,000,000 hydrogen atoms. The mass of an atom of gold is 0.000 000 000 000 000 000 000 327 gram. Writing and using such large and small numbers is very cumbersome. You can work more easily with these numbers by writing them in scientific, or exponential, notation.

In scientific notation, a given number is written as the product of two numbers: a coefficient and 10 raised to a power. For example, the number 602,000,000,000,000,000,000,000 written in scientific notation is \(6.02 \times 10^{23}\). Figure 3.1 illustrates how to express the number of stars in a galaxy by using scientific notation. For more practice on writing numbers in scientific notation, refer to page R56 of Appendix C.

Figure 3.1 Expressing very large numbers, such as the estimated number of stars in a galaxy, is easier if scientific notation is used.

Guide for Reading

Key Concepts

- How do measurements relate to science?
- How do you evaluate accuracy and precision?
- Why must measurements be reported to the correct number of significant figures?
- How does the precision of a calculated answer compare to the precision of the measurements used to obtain it?

Vocabulary

- measurement
- scientific notation
- accuracy
- precision
- accepted value
- experimental value
- error
- percent error
- significant figures

Reading Strategy

Building Vocabulary

As you read, write a definition of each vocabulary term in your own words.

2

INSTRUCT

Connecting to Your World

Have students study the photograph and read the text that opens the section. Ask, How do you think scientists ensure measurements are accurate and precise? (Acceptable answers include that scientists make multiple measurements by using the most precise equipment available. They use samples with known values to check the reliability of the equipment.)
Using and Expressing Measurements

Figure 3.1 Have students study the photograph and read the text that opens the section. Ask: How is the exponent of a number expressed in scientific notation related to the number of places the decimal point is moved to the left in a number larger than 1? (They are equal.)

Accuracy, Precision, and Error

Precision and Accuracy

Purpose To illustrate the concepts of precision and accuracy.

Materials a small object (such as a lead fishing weight), triple-beam balance

Procedure Place the object and a triple-beam balance in a designated area. Set a deadline by which each student will have measured the mass of the object. After everyone has had an opportunity, have students compile a summary of all the measurements. Illustrate precision by having the students find the average and compare their measurement to it.

Expected Outcome Measured values should be similar, but not necessarily identical for all students.

FYI Other analogies that may be useful in explaining precision vs. accuracy:
• casting a fishing line
• pitching horseshoes
• a precision marching band

Accuracy, Precision, and Error

Your success in the chemistry lab and in many of your daily activities depends on your ability to make reliable measurements. Ideally, measurements should be both correct and reproducible.

Accuracy and Precision Consistency and reproducibility relate to the concepts of accuracy and precision, two words that mean the same thing to many people. In chemistry, however, their meanings are quite different. Accuracy is a measure of how close a measurement comes to the actual or true value of whatever is measured. Precision is a measure of how close a series of measurements are to one another. To evaluate the accuracy of a measurement, the measured value must be compared to the correct value. To evaluate the precision of a measurement, you must compare the values of two or more repeated measurements.

Darts on a dartboard illustrate accuracy and precision in measurement. Let the bull’s-eye of the dartboard represent the true, or correct, value of what you are measuring. The closeness of a dart to the bull’s-eye corresponds to the degree of accuracy. The closer it comes to the bull’s-eye, the more accurately the dart was thrown. The closeness of several darts to one another corresponds to the degree of precision. The closer together the darts are, the greater the precision and the reproducibility.

Look at Figure 3.2 as you consider the following outcomes.

a. All of the darts land close to the bull’s-eye and to one another. Closeness to the bull’s-eye means that the degree of accuracy is great. Each dart in the bull’s-eye corresponds to an accurate measurement of a value.

b. All of the darts land close to one another but far from the bull’s-eye. The precision is high because of the closeness of grouping and thus the high level of reproducibility. The results are inaccurate, however, because of the distance of the darts from the bull’s-eye.

c. The darts land far from one another and from the bull’s-eye. The results are both inaccurate and imprecise.

Checkpoint How does accuracy differ from precision?

Facts and Figures

Striving for Scientific Accuracy

The French chemist, Antoine Lavoisier, worked hard to establish the importance of accurate measurement in scientific inquiry. Lavoisier devised an experiment to test the Greek scientists’ idea that when water was heated, it could turn into earth. For 100 days, Lavoisier boiled water in a glass flask constructed to allow steam to condense without escaping. He weighed the water and the flask separately before and after boiling. He found that the mass of the water had not changed. The flask, however, lost a small mass equal to the sediment he found in the bottom of it. Lavoisier proved that the sediment was not earth, but part of the flask etched away by the boiling water.
Determining Error  Note that an individual measurement may be accurate or inaccurate. Suppose you use a thermometer to measure the boiling point of pure water at standard pressure. The thermometer reads 99.1°C. You probably know that the true or accepted value of the boiling point of pure water under these conditions is actually 100.0°C. There is a difference between the accepted value, which is the correct value based on reliable references, and the experimental value, the value measured in the lab. The difference between the experimental value and the accepted value is called the error.

\[
\text{Error} = \text{experimental value} - \text{accepted value}
\]

Error can be positive or negative depending on whether the experimental value is greater than or less than the accepted value.

For the boiling-point measurement, the error is 99.1°C – 100.0°C, or = 0.9°C. The magnitude of the error shows the amount by which the experimental value differs from the accepted value. Often, it is useful to calculate the relative error, or percent error. The percent error is the absolute value of the error divided by the accepted value, multiplied by 100%.

\[
\text{Percent error} = \left(\frac{\text{error}}{\text{accepted value}}\right) \times 100\%
\]

Using the absolute value of the error means that the percent error will always be a positive value. For the boiling-point measurement, the percent error is calculated as follows.

\[
\text{Percent error} = \left(\frac{0.9°C}{100.0°C}\right) \times 100\% = 0.9\%
\]

Just because a measuring device works doesn’t necessarily mean that it is accurate. As Figure 3.3 shows, a weighing scale that does not read zero when nothing is on it is bound to yield error. In order to weigh yourself accurately, you must first make sure that the scale is zeroed.

Word Origins  The phrase per annum means “by the year.”

Percent comes from the Latin words per, meaning “by” or “through,” and centum, meaning “100.”

Use Visuals  Figure 3.2 Have students inspect Figure 3.2. Ask, If one dart in Figure 3.2c were closer to the bull’s-eye, what would happen to the accuracy? (The accuracy would increase.) What would happen to the precision? (The precision would increase.) What is the operational definition of error implied by this figure? (The error is the distance between the dart and the bull’s-eye.)

Discuss  Review the concept of absolute value. Ask, What is the meaning of a positive error? (Measured value is greater than accepted value.) What is the meaning of a negative error? (Measured value is less than accepted value.) Explain that the absolute value of the error is a positive value that describes the difference between the measured value and the accepted value, but not which is greater.

Answers to...  Figure 3.3  Ask, What is the meaning of a positive error? (Measured value is greater than accepted value.) What is the meaning of a negative error? (Measured value is less than accepted value.) Explain that the absolute value of the error is a positive value that describes the difference between the measured value and the accepted value, but not which is greater.
Significant Figures in Measurements

Discuss
Point out that the concept of significant figures applies only to measured quantities. If students ask why an estimated digit is considered significant, tell them a significant figure is one that is known to be reasonably reliable. A careful estimate fits this definition.

FYI
When calibration marks on an instrument are spaced very close together (e.g., on certain thermometers and graduated cylinders), it is sometimes more practical to estimate a measurement to the nearest half of the smallest calibrated increment, rather than to the nearest tenth.

CLASS Activity

Olympic Times
Purpose To illustrate how similar measurements from different eras may vary in precision
Materials Almanacs or Internet access
Procedure Have students look up the winning times for the men’s and women’s 100-meter dashes at the 1948 and 2000 Olympic Games. Then have them answer the following question.
Why do the more recently recorded race times contain more digits to the right of the decimal? (Because the technology used for timekeeping improved to allow for more precise measurements.)
Expected Outcome Students should find that the race times from 1948 were recorded to the nearest tenth of a second. The race times from 2000 were recorded to the nearest hundredth of a second.

Significant Figures in Measurements
Supermarkets often provide scales like the one in Figure 3.4. Customers use these scales to measure the weight of produce that is priced per pound. If you use a scale that is calibrated in 0.1-lb intervals, you can easily read the scale to the nearest tenth of a pound. With such a scale, however, you can also estimate the weight to the nearest hundredth of a pound by noting the position of the pointer between calibration marks.

Suppose you estimate a weight that lies between 2.4 lb and 2.5 lb to be 2.46 lb. The number in this estimated measurement has three digits. The first two digits in the measurement (2 and 4) are known with certainty. But the rightmost digit (6) has been estimated and involves some uncertainty. These three reported digits all convey useful information, however, and are called significant figures. The significant figures in a measurement include all of the digits that are known, plus a last digit that is estimated. Measurements must always be reported to the correct number of significant figures because calculated answers often depend on the number of significant figures in the values used in the calculation.

Instruments differ in the number of significant figures that can be obtained from their use and thus in the precision of measurements. The three meter sticks in Figure 3.5 can be used to make successively more precise measurements of the board.

Rules for determining whether a digit in a measured value is significant:

1. Every nonzero digit in a reported measurement is assumed to be significant. The measurements 24.7 meters, 0.743 meter, and 714 meters each express a measure of length to three significant figures.

2. Zeros appearing between nonzero digits are significant. The measurements 7003 meters, 40.79 meters, and 1.503 meters each have four significant figures.

3. Leftmost zeros appearing in front of nonzero digits are not significant. They act as placeholders. The measurements 0.0071 meter, 0.42 meter, and 0.000 999 meter each have only two significant figures. The zeros to the left are not significant. By writing the measurements in scientific notation, you can eliminate such placeholder zeros: in this case, $7.1 \times 10^{-2}$ meter, $4.2 \times 10^{-1}$ meter, and $9.9 \times 10^{-5}$ meter.

4. Zeros at the end of a number and to the right of a decimal point are always significant. The measurements 43.00 meters, 1.010 meters, and 9.000 meters each have four significant figures.
Use Visuals

Figure 3.5 As students inspect Figure 3.5, model the use of meter stick A by pointing out that one can be certain that the length of the board is between 0 and 1 m, and one can say that the actual length is closer to 1 m. Thus, one can estimate the length as 0.6 m. Similarly, using meter stick B, one can say with certainty that the length is between 60 and 70 cm. Because the length is very close to 60 cm, one should estimate the length as 61 cm or 0.61 m. Have students study meter stick C and use similar reasoning to describe the measurement and estimation process. Ask, “If meter stick C were divided into 0.001-m intervals, as are most meter sticks, what would be the estimated length of the board in meters?” (Acceptable answers range from 0.6065 to 0.6074 m). In millimeters? (606.5 to 607.4 mm)

Discuss

Be sure to review the role of zeros in determining the number of significant figures. When adding or subtracting numbers expressed in scientific notation, remind students that the numbers must all have the same exponent.

Differentiated Instruction

Less Proficient Readers

Have students write in their own words the rules for determining the number of significant digits. Help them if necessary. Have them measure several lengths, masses, and volumes; then have them use their rules to determine the correct number of significant figures for each measurement.

Answers to...

Figure 3.5 The measurements are:

a. 0.6 m (1 significant figure),
b. 0.61 m (2 significant figures),
c. 0.607 m (3 significant figures)
CONCEPTUAL PROBLEM 3.1

Counting Significant Figures in Measurements

How many significant figures are in each measurement?

1. 12.3 cm
2. 40,506 mm
3. 1.200 × 10^3 m
4. 22 meter sticks
5. 0.07080 m
6. 91,000 m

### Analyze
Identify the relevant concepts.

- The location of each zero in the measurement and the location of the decimal point determine which of the rules apply.
- Significant figures.

### Solve
Apply the concepts to this problem.

- All nonzero digits are significant (rule 1).
- Use rules 2 through 6 to determine if the zeros are significant.

1. a. three (rule 1)
2. b. five (rule 2)
3. c. five (rule 4)
4. d. unlimited (rule 6)
5. e. four (rules 2, 3, 4)
6. f. two (rule 5)

### Practice Problems

1. Count the significant figures in each length.
   a. 0.05730 meters
   b. 8765 meters
   c. 0.00073 meters
   d. 40.007 meters

2. How many significant figures are in each measurement?
   a. 145 grams
   b. 0.074 meter
   c. 8.750 × 10^-2 gram
   d. 1.072 meter

### Significant Figures in Calculations

Suppose you use a calculator to find the area of a floor that measures 7.7 meters by 5.4 meters. The calculator would give an answer of 41.58 square meters. The calculated area is expressed to four significant figures. However, each of the measurements used in the calculation is expressed to only two significant figures. So the answer must also be reported to two significant figures (42 m^2).

In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated. The calculated value must be rounded to make it consistent with the measurements from which it was calculated.

### Rounding
To round a number, you must first decide how many significant figures the answer should have. This decision depends on the given measurements and on the mathematical process used to arrive at the answer. Once you know the number of significant figures your answer should have, round to that many digits, counting from the left. If the digit immediately to the right of the last significant digit is less than 5, it is simply dropped and the value of the last significant digit stays the same. If the digit in question is 5 or greater, the value of the digit in the last significant place is increased by 1.

### Checkpoint

Why must a calculated answer generally be rounded?
Sample Problem 3.1

Rounding Measurements

Round off each measurement to the number of significant figures shown in parentheses. Write the answers in scientific notation.

1. 314.721 meters (four)
2. 0.001 775 meter (two)
3. 8792 meters (two)

Analyze Identify the relevant concepts.

Round off each measurement to the number of significant figures indicated. Then apply the rules for expressing numbers in scientific notation.

Solve Apply the concepts to this problem.

Count from the left and apply the rule to the digit immediately to the right of the digit to which you are rounding. The arrow points to the digit immediately following the last significant digit.

a. 314.7 meters
   - 2 is less than 5, so you do not round up.
   - 314.7 meters = 3.147 \times 10^2 meters

b. 0.001 775 meter
   - 7 is greater than 5, so round up.
   - 0.001 775 meter = 1.8 \times 10^{-3} meter

c. 8792 meters
   - 9 is greater than 5, so round up.
   - 8792 meters = 8.8 \times 10^3 meters

Evaluate Do the results make sense?

The rules for rounding and for writing numbers in scientific notation have been correctly applied.

Practice Problems

3. Round each measurement to three significant figures. Write your answers in scientific notation.
   a. 87.073 meters
   b. 4.3621 \times 10^9 meters
   c. 0.01552 meter
   d. 9009 meters
   e. 1.7777 \times 10^{-3} meter
   f. 629.55 meters

4. Round each measurement in Practice Problem 3 to one significant figure. Write each of your answers in scientific notation.

For help with scientific notation, go to page R56.
Section 3.1 (continued)

Discuss

The rules for rounding calculated numbers can be compared with the old adage, "A chain is only as strong as its weakest link." Explain that an answer cannot be more precise than the least precise value used to calculate the answer. Ask, In addition and subtraction, what is the least precise value? (The measurement with the fewest digits to the right of the decimal point.) In multiplication and division, what is the least precise value? (The measurement with the fewest significant figures.) If students wonder why addition and subtraction rules differ from multiplication and division rules, point out that in addition and subtraction of measurements, the measurements are of the same property, such as length or volume. However, in the multiplication and division of measurements, new quantities or properties are being described, such as speed (length ÷ time), area (length × length), and density (mass ÷ volume).

Sample Problem 3.2

Answers

5. a. 79.2 m  b. 7.33 m  c. 11.53 m  d. 17.3 m
6. 23.8 g

Practice Problems Plus

Find the total mass of three diamonds that have masses of 14.2 grams, 8.73 grams, and 0.912 gram. (147.72 g)

Sample Problem 3.3

Answers

7. a. \(1.8 \times 10^3\) m\(^3\)  b. \(6.75 \times 10^2\) m\(^3\)  c. \(5.87 \times 10^{-3}\) min
8. \(1.3 \times 10^3\) m\(^3\)

Practice Problems Plus

Calculate the volume of a house that has dimensions of 12.52 meters by 36.86 meters by 2.46 meters. (1.14 \(\times 10^3\) m\(^3\))
Multiplication and Division: In calculations involving multiplication and division, you need to round the answer to the same number of significant figures as the measurement with the least number of significant figures. The position of the decimal point has nothing to do with the rounding process when multiplying and dividing measurements. The position of the decimal point is important only in rounding the answers of addition or subtraction problems.

Check point

How many significant figures must you round an answer to when performing multiplication or division?

SAMPLE PROBLEM 3.3

Significant Figures in Multiplication and Division

Perform the following operations. Give the answers to the correct number of significant figures.

a. 7.55 meters × 0.34 meter
b. 2.10 meters ÷ 0.70 meter
c. 2.4526 meters ÷ 8.4

1. Analyze Identify the relevant concepts.

Perform the required math operation and then analyze each of the original numbers to determine the correct number of significant figures required in the answer.

2. Solve Apply the concepts to this problem.

Round the answers to the match the measurement with the least number of significant figures.

a. 7.55 meters × 0.34 meter = 2.567 (meter)² = 2.6 meters²
b. 2.10 meters ÷ 0.70 meter = 1.47 (meter)² = 1.5 meters²
c. 2.4526 meters ÷ 8.4 = 0.291 976 meter = 0.29 meter

3. Evaluate Do the results make sense?

The mathematical operations have been performed correctly, and the resulting answers are reported to the correct number of places.

Practice Problems

7. Solve each problem. Give your answers to the correct number of significant figures and in scientific notation.

a. 8.3 meters × 2.22 meters
b. 0.8427 meters ÷ 12.5
c. 35.2 seconds ÷ 5 minute

8. Calculate the volume of a warehouse that has inside dimensions of 22.4 meters by 11.3 meters by 5.2 meters.

(Volume = 1 × l × w × h)

Answers to...

Check point

The same number of significant figures as the measurement with the least number of significant figures.

Scientific Measurement 71
Section 3.1 (continued)

1. ASSESS

Evaluate Understanding
Write the following sets of measurements on the board.
(1) 78°C, 76°C, 75°C
(2) 77°C, 78°C, 78°C
(3) 60°C, 81°C, 82°C
Ask: If these sets of measurements were made of the boiling point of a liquid under similar conditions, explain which set is the most precise? (Set 2 is the most precise because the three measurements are closest together.) What would have to be known to determine which is the most accurate? (the accepted value of the liquid’s boiling point)

Reteach
Use Figure 3.5 to reteach the method of correctly recording the number of significant figures in a measurement. Then have students convert each measurement into scientific notation.

3.1 Section Assessment

9. **Key Concept** How do measurements relate to experimental science?
10. **Key Concept** How are accuracy and precision evaluated?
11. **Key Concept** Why must a given measurement always be recorded to the correct number of significant figures?
12. **Key Concept** How does the precision of a calculated answer compare to the precision of the measurements used to obtain it?
13. A technician experimentally determined the boiling point of octane to be 125.7°C. The actual boiling point of octane is 125.7°C. Calculate the error and the percent error.
14. Determine the number of significant figures in each of the following.
   a. 11 soccer players
   b. 0.070020 meter
   c. 10,800 meters
   d. 5.00 cubic meters
15. Solve the following and express each answer in scientific notation and to the correct number of significant figures.
   a. (5.3 × 10^4) + (1.5 × 10^4)
   b. (7.2 × 10^5) + (1.8 × 10^5)
   c. 10^6 × 10^5 × 10^8
   d. (9.12 × 10^-4) - (4.7 × 10^-4)
   e. (5.4 × 10^-2) × (3.5 × 10^-4)

Quick LAB

Accuracy and Precision

**Purpose**
To measure the dimensions of an object as accurately and precisely as possible and to apply rules for rounding answers calculated from the measurements.

**Materials**
• 5 metric index cards
• metric ruler

**Procedure**
1. Use a metric ruler to measure the length and width of an index card as accurately and precisely as you can. The hundredths place in your measurement should be estimated.
2. Calculate the perimeter (2 × (length + width)) and the area (length × width) of the index card. Write both your unrounded answers and your correctly rounded answers on the chalkboard.

**Analyze and Conclude**
1. How may significant figures are in your measurements of length and of width?
2. How do your measurements compare with those of your classmates?
3. How many significant figures are in your calculated value for the area? In your calculated value for the perimeter? Do your rounded answers have as many significant figures as your classmates’ measurements?
4. Assume that the correct (accurate) length and width of the card are 12.75 cm and 7.62 cm, respectively. Calculate the percent error for each of your two measurements.

If your class subscribes to the Interactive Textbook, use it to review key concepts in Section 3.1.

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