14.3 Ideal Gases

**Connect to Your World**
Solid carbon dioxide, or dry ice, is used to protect products that need to be kept cold during shipping. The adjective dry refers to a key advantage of shipping with dry ice. Dry ice doesn’t melt. It sublimes. Dry ice can exist because gases don’t obey the assumptions of kinetic theory at all conditions. In this section, you will learn how real gases differ from the ideal gases on which the gas laws are based.

**Ideal Gas Law**
With the combined gas law, you can solve problems with three variables: pressure, volume, and temperature. The combined gas law assumes that the amount of gas does not vary. You cannot use the combined gas law to calculate the number of moles of a gas in a fixed volume at a known temperature and pressure. To calculate the number of moles of a contained gas requires an expression that contains the variable $n$. The combined gas law can be modified to include the number of moles.

The number of moles of gas is directly proportional to the number of particles. The volume occupied by a gas at a specified temperature and pressure also must depend on the number of particles. So moles must be directly proportional to volume as well. You can introduce moles into the combined gas law by dividing each side of the equation by $n$.

$$\frac{P}{n} = \frac{V}{n} \times \frac{1}{n}$$

This equation shows that $(P \times V)/(n \times n)$ is a constant. This constant holds for ideal gases—gases that conform to the gas laws. If you know the values for $P$, $V$, $T$, and $n$ for one set of conditions, you can calculate a value for the constant. Recall that 1 mol of every gas occupies 22.4 L at STP (101.3 kPa and 273 K). You can use these values to find the value of the constant, which has the symbol $R$ and is called the ideal gas constant. Insert the values of $P$, $V$, $T$, and $n$ into $(P \times V)/(n \times n)$.

$$R = \frac{P \times V}{n \times T} = \frac{101.3 \text{ kPa} \times 22.4 \text{ L}}{273 \text{ K} \times 1} = 8.31 \text{ L} \cdot \text{kPa}/(\text{K} \cdot \text{mol})$$

The ideal gas constant ($R$) has the value 8.31 (L·kPa)/(K·mol). The gas law that includes all four variables—$P$, $V$, $T$, and $n$—is called the ideal gas law. It is usually written as follows.

$$PV = nRT$$

### Section Resources

**Print**
- Guided Reading and Study Workbook, Section 14.3
- Core Teaching Resources, Section 14.3
- Review, Interpreting Graphics
- Transparencies, T156–T157
- Probeware Laboratory Manual, Section 14.3

**Technology**
- Interactive Textbook with ChemASAP, Problem Solving 14.24, Assessment 14.3
- Virtual Chemistry Labs, 13, 14

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**FOCUS**

**Objectives**
14.3.1 Compute the value of an unknown using the ideal gas law.
14.3.2 Compare and contrast real and ideal gases.

**Guide for Reading**

### Build Vocabulary

**Word Parts**
Have students look up the meanings of *ideal* and *real*. Have them write definitions that include the terms theoretical and actual.

### Reading Strategy

**Make Inferences**
Have students use their understanding of the terms real and ideal to infer the differences between real and ideal gases. As the students read the section, have them evaluate and revise their inference.

**INSTRUCT**

Have students study the photograph and read the text that opens the section. Ask, How does the existence of dry ice violate the assumptions of the kinetic theory? (The kinetic theory assumes that there are no attractions among the particles in a gas. But there must be attractions between carbon dioxide molecules for the gas to solidify.)

### Ideal Gas Law

**Discuss**
Sketch two containers with identical volumes on the board. Tell students one container is filled with neon gas and the other with helium at the same temperature and pressure. Ask students to use the ideal gas law to find the equation for the number of moles in each container. Students should determine that $n_{\text{Ne}} = n_{\text{He}}$.

### Reading Strategy

**After you read this section, explain the difference between ideal and real gases must be attractions between carbon dioxide molecules for the gas to solidify.**

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**14.3.2 Compare and contrast real and ideal gases.**

Have students study the photograph and read the text that opens the section. Ask, How does the existence of dry ice violate the assumptions of the kinetic theory? (The kinetic theory assumes that there are no attractions among the particles in a gas. But there must be attractions between carbon dioxide molecules for the gas to solidify.)

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**Technology**
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- Virtual Chemistry Labs, 13, 14
Sample Problem 14.5

Using the Ideal Gas Law to Find the Amount of a Gas

A deep underground cavern contains $2.24 \times 10^6$ L of methane gas (CH$_4$) at a pressure of $1.50 \times 10^3$ kPa and a temperature of 315 K. How many kilograms of CH$_4$ does the cavern contain?

1. **Analyze** List the knowns and the unknown.

   Knowns:  
   - $P = 1.50 \times 10^3$ kPa  
   - $V = 2.24 \times 10^6$ L  
   - $T = 315$ K  
   - $R = 8.31$ (L·kPa)/(K·mol)  
   - molar mass$_{CH_4} = 16.0$ g

   Unknown:  
   - ? kg CH$_4$

   Calculate the number of moles ($n$) using the ideal gas law. Use the molar mass to convert moles to grams. Then convert grams to kilograms.

2. **Calculate** Solve for the unknown.

   Rearrange the equation for the ideal gas law to isolate $n$.

   $$n = \frac{PV}{RT}$$

   Substitute the known quantities into the equation to find the number of moles of methane.

   $$n = \frac{(1.50 \times 10^3 \text{ kPa}) \times (2.24 \times 10^6 \text{ L})}{8.31 \frac{\text{L·kPa}}{\text{K·mol}} \times 315 \text{ K}} = 1.28 \times 10^6 \text{ mol CH}_4$$

   A mole-mass conversion gives the number of grams of methane.

   $$1.28 \times 10^6 \text{ mol CH}_4 \times \frac{16.0 \text{ g CH}_4}{1 \text{ mol CH}_4} = 20.5 \times 10^6 \text{ g CH}_4 = 2.05 \times 10^7 \text{ kg CH}_4$$

3. **Evaluate** Does the result make sense?

   Although the methane is compressed, its volume is still very large. So it is reasonable that the cavern contains a large mass of methane.

Practice Problems

23. When the temperature of a rigid hollow sphere containing 685 L of helium gas is held at 621 K, the pressure of the gas is $1.89 \times 10^5$ kPa. How many moles of helium does the sphere contain?

24. A child’s lungs can hold 2.20 L. How many grams of air do her lungs hold at a pressure of 102 kPa and a body temperature of 37°C? Use a molar mass of 29 g for air, which is about 20% O$_2$ (32 g/mol) and 80% N$_2$ (28 g/mol).
Section 14.3 (continued)

Ideal Gases and Real Gases

Carbon Dioxide from Antacid Tablets

Objective After completing this activity, students will be able to:

• measure the amount of carbon dioxide gas given off when antacid tablets dissolve in water.

Skills Focus Observing, Calculating, Measuring

Prep Time 10 minutes  
Class Time 40 minutes

Safety If you use latex balloons, check to see if any students are allergic to latex.

Expected Outcome The volumes of CO\(_2\) produced will reflect the amount of antacid each balloon contains.

Analyze and Conclude

1. The volume of the balloon is directly proportional to the number of tablets.
2. Answers will vary, but the masses and numbers of moles should be in ratios of 1:2:3 for the three balloons.
3. Possible answer: 2.0 g of NaHCO\(_3\) (molar mass = 84.01 g) should yield about 1.2 \times 10^{-2} \text{ mol} of CO\(_2\).

For Enrichment

Have students use a similar procedure to compare different brands of effervescent antacids instead of different amounts of the same antacid.

Ideal Gases and Real Gases

An ideal gas is one that follows the gas laws at all conditions of pressure and temperature. Such a gas would have to conform precisely to the assumptions of kinetic theory. Its particles could have no volume, and there could be no attraction between particles in the gas. As you probably suspect, there is no gas for which these assumptions are true. So an ideal gas does not exist. Nevertheless, at many conditions of temperature and pressure, real gases behave very much like an ideal gas.

The particles in a real gas do have volume, and there are attractions between the particles. Because of these attractions, a gas can condense, or even solidify, when it is compressed or cooled. For example, if water vapor is cooled below 100°C at standard atmospheric pressure, it condenses to a liquid. The behavior of other real gases is similar, although lower temperatures and greater pressures may be required. Such conditions are required to produce the liquid nitrogen in Figure 14.14. Real gases differ most from an ideal gas at low temperatures and high pressures.

Analyze and Conclude

1. Make a graph of volume versus number of tablets. Use your graph to describe the relationship between the number of tablets used and the volume of the balloon.
2. Assume that the balloon is filled with carbon dioxide gas at 20°C and standard pressure. Calculate the mass and the number of moles of CO\(_2\) in each balloon at maximum inflation.
3. If a typical antacid tablet contains 2.0 g of sodium hydrogen carbonate, how many moles of CO\(_2\) should one tablet yield? Compare this theoretical value with your results.

Figure 14.14 is this flask used to store liquid nitrogen, there are two walls with a vacuum in between.
Figure 14.15 shows how the value of the ratio \(\frac{PV}{nRT}\) changes as pressure increases. For an ideal gas, the result is a horizontal line because the ratio is always equal to 1. For real gases at high pressure, the ratio may deviate, or depart, from the ideal. When the ratio is greater than 1, the curve rises above the ideal gas line. When the ratio is less than 1, the curve drops below the line. The deviations can be explained by two factors. As attractive forces reduce the distance between particles, a gas occupies less volume than expected, causing the ratio to be less than 1. But the actual volume of the molecules causes the ratio to be greater than 1.

In portions of the curves below the line, intermolecular attractions dominate. In portions of the curves above the line, molecular volume dominates. Look at the curves for methane \((CH_4)\) at 0°C and at 200°C. At 200°C, the molecules have more kinetic energy to overcome intermolecular attractions. So the curve for \(CH_4\) at 200°C never drops below the line.

### 14.3 Section Assessment

25. **Key Concept** What do you need to calculate the amount of gas in a sample at given conditions of temperature, pressure, and volume?

26. **Key Concept** Under what conditions do real gases deviate most from ideal behavior?

27. What is an ideal gas?

28. Determine the volume occupied by 0.382 mol of a gas at 15°C if the pressure is 81.8 kPa.

29. What pressure is exerted by 0.450 mol of a gas at 25°C if the gas is in a 0.500-L container?

30. Use the kinetic theory of gases to explain this statement: No gas exhibits ideal behavior at all temperatures and pressures.

### Interpreting Graphs

- **Observing** What are the values of \(\frac{PV}{nRT}\) for an ideal gas at 20,000 and 60,000 kPa?
- **Comparing** What variable is responsible for the differences between the two \((CH_4)\) curves?
- **Making Generalizations** How does an increase in pressure affect the \(\frac{PV}{nRT}\) ratio for real gases?

### Use Visuals

**Figure 14.15** Point out that the straight line represents ideal conditions. For an ideal gas, by definition \(PV = nRT\), and the ratio of \(PV\) to \(nRT\) is 1. When the volume of a gas is greater than expected, the ratio tends to be greater than 1. When the volume is less than expected, the ratio tends to be less than 1. The main factors that affect the volume are intermolecular attractions and the actual volume of the particles.

### Assess

**Evaluate Understanding** Have students apply the ideal gas law to a sample of gas in a balloon. Ask them to explain why in a balloon \(n\) and \(A\) are constants and \(P, V,\) and \(T\) are variables.

**Re-teach** Point out to students that one advantage of the ideal gas law is that it enables them to find the number of moles of a gas by measuring its temperature, pressure, and volume.

**Connecting Concepts** The nitrogen molecule is nonpolar and the ammonia molecule is polar. So there are stronger intermolecular attractions in ammonia.

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**Section 14.3 Assessment**

25. an expression that contains the variable \(n\)

26. Real gases deviate from ideal behavior at low temperatures and high pressures.

27. An ideal gas is a gas that follows the gas laws at all conditions of pressure and temperature.

28. 17.0 L

29. \(1.71 \times 10^3\) kPa

30. In real gases, there are attractions between molecules, and the molecules have volume. At low temperatures, attractions between molecules pull them together and reduce the volume. At high pressures, the volume occupied by the molecules is a significant part of the total volume.

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**Introduction to Textbook** If your class subscribes to the Interactive Textbook, use it to review key concepts in Section 14.3.
Decompression sickness is an application of Henry’s law (which is discussed in Chapter 16): At a given temperature, the solubility of a gas in a liquid is directly proportional to the pressure of the gas. Dissolved nitrogen is more problematic than dissolved oxygen because oxygen released during decompression can be removed from the blood and used by the cells. Nitrogen is not used up by the cells and must be excreted through the lungs. Varying the composition of the compressed gas and using dive charts are two strategies for combating decompression sickness.

Discuss
Discuss the content of the article in the context of Dalton’s law of partial pressures. Remind students that, according to Dalton’s law of partial pressures, the total pressure exerted by a mixture of gases is equal to the sum of the partial pressures exerted by each of the different gases in the mixture. The fractional contribution to pressure exerted by each gas does not change as the temperature, pressure, or volume changes as long as the composition of the mixture is constant. Ask students to use Dalton’s law of partial pressures to explain how changing the mixture of gases in the tanks used by divers can help prevent problems.

CLASS Activity
Effect of Depth on Partial Pressures
With Table 14.1 displayed on an overhead projector, point out that the air at sea level is about 21% oxygen and 78% nitrogen. The values for the partial pressures given in Table 14.1 reflect conditions at 1 atm or 101.3 kPa. Have students calculate the partial pressures of each gas that a scuba diver, breathing the same air mixture, would experience at depths of 100 feet (approximately 4 atm) and 300 feet (approximately 10 atm).

Facts and Figures
Decompression Sickness
The invention of compressed air in the 1840s allowed people to work under water, with one major drawback. Decompression sickness was initially called caisson disease because workers constructing the Brooklyn Bridge worked in water-tight containers called caissons. The pressure of the air in the caissons needed to be greater than atmospheric pressure to withstand the pressure of the surrounding water. The condition was also called “the bends.” In 1878, Paul Bert stated that workers could avoid the bends if they ascended gradually to the surface. Bert referred to the work of Robert Boyle. In 1667, Boyle observed a bubble form in the eye of a viper that was placed in a compressed atmosphere and then removed. (Boyle reported that the viper appeared distressed by the experience.)
Dive tables help a diver avoid decompression sickness. Divers use the data to control the length and frequency of their dives, and the rate at which they return to the surface.

**Table 1: End of dive letter group**

At the end of a dive, the diver matches the time and depth of the dive to a letter group. Letter A represents the least amount of nitrogen left in the blood after the dive. The circled numbers show the maximum time a diver can spend at a given depth without having to make a decompression stop during the ascent.

**Table 2: Surface interval time**

The longer a diver spends at the surface, the more nitrogen is excreted through the lungs. A diver can move from Group H to Group A after 8 hours at the surface.

**Table 3: Repetitive dive timetable**

Divers use this table to determine an adjusted maximum dive time before doing a second dive. They also use the table to determine a letter group at the end of the second dive.

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**Use Visuals**

Discuss the purpose of dive tables and explain how to use them. Further explanation of the tables is provided at the NAUI website. Have students use the indicated table to answer the following questions.

**Table 1**

1. How long can a diver stay at 24 m without needing to make a decompression stop? (35 minutes)
2. A diver stays at a depth of 33 m for 20 minutes. Will she have to make a decompression stop when returning to the surface? Explain. (Yes, the maximum time with no decompression at this depth is 15 minutes.)

**Table 2**

Explain that after a dive, a diver is assigned a letter based on the depth and time of the dive. This letter determines how soon the diver can dive again. The higher the letter, the longer it takes for nitrogen to be removed from the blood.

1. A diver assigned to Group G has been out of the water for 4 hours. What group is he now in? (Group C)
2. After how many hours has all nitrogen that might be harmful been eliminated from the blood? (24 hours)

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**Gifted and Talented**

Have students find out how to use the repetitive dive table (Table 3). Have them teach other students how to use the table and provide problems for the students to solve.

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**Answers to...**

Interpreting Diagrams

Oxygen is essential; nitrogen is optional.

The Behavior of Gases