Coordinate Algebra  Unit 3: Linear and Exponential Functions

<table>
<thead>
<tr>
<th>Standards</th>
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<tbody>
<tr>
<td><strong>Represent and solve equations and inequalities graphically</strong></td>
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**MCC9-12.A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*

**MCC9-12.A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Understand the concept of a function and use function notation**

**MCC9-12.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \). *(Draw examples from linear and exponential functions.)*

**MCC9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

**MCC9-12.F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

**Interpret functions that arise in applications in terms of the context**

**MCC9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *(Focus on linear and exponential functions.)*

**MCC9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

**MCC9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

**Analyze functions using different representations**

**MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

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<tbody>
<tr>
<td>In this unit students will:</td>
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<tr>
<td>• Represent and solve linear equations and inequalities graphically using real-world contexts.</td>
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<tr>
<td>• Use function notation.</td>
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<tr>
<td>• Interpret linear and exponential functions and model how different representations may be used based on the situation presented.</td>
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<td>• Build a function to model a relationship between two quantities.</td>
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<td>• Create new functions from existing functions.</td>
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<tr>
<td>• Construct and compare linear and exponential models and solve problems.</td>
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<td>• Interpret expressions for functions in terms of the situation they model.</td>
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<tr>
<td>MCC9-12.F.IF.7a</td>
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<tr>
<td>MCC9-12.F.IF.7c</td>
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<td>MCC9-12.F.IF.9</td>
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**Build a function that models a relationship between two quantities**

<table>
<thead>
<tr>
<th>MCC9-12.F.BF.1</th>
<th>Write a function that describes a relationship between two quantities. <em>(Limit to linear and exponential functions.)</em></th>
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<tr>
<td>MCC9-12.F.BF.1a</td>
<td>Determine an explicit expression, a recursive process, or steps for calculation from a context. <em>(Limit to linear and exponential functions.)</em></td>
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<tr>
<td>MCC9-12.F.BF.1b</td>
<td>Combine standard function types using arithmetic operations. <em>(Limit to linear and exponential functions.)</em></td>
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<tr>
<td>MCC9-12.F.BF.2</td>
<td>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</td>
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</table>

**Build new functions from existing functions**

| MCC9-12.F.BF.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its $y$-intercept.)* |

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<thead>
<tr>
<th>MCC9-12.F.LE.1</th>
<th>Distinguish between situations that can be modeled with linear functions and with exponential functions.</th>
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<tr>
<td>MCC9-12.F.LE.1a</td>
<td>Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.</td>
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<tr>
<td>MCC9-12.F.LE.1b</td>
<td>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
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<tr>
<td>MCC9-12.F.LE.1c</td>
<td>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
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<tr>
<td>MCC9-12.F.LE.2</td>
<td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td>
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<tr>
<td>MCC9-12.F.LE.3</td>
<td>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</td>
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<tr>
<td>MCC9-12.F.LE.5</td>
<td>Interpret the parameters in a linear or exponential function in terms of a context. <em>(Limit exponential functions to those of the form $f(x) = b^x + k$).</em></td>
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STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear and exponential functions that arise in applications in terms of the context.
- When analyzing linear and exponential functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- New functions can be created from existing functions.
- Understand how to construct and compare linear and exponential models and solve problems.
- Understand how to interpret expressions for functions in terms of the situation they model.

SELECTED TERMS AND SYMBOLS

- Arithmetic Sequence.
- Average Rate of Change
- Coefficient
- Common difference
- Common ratio
- Continuous
- Constant Rate of Change
- Discrete
- Domain
- End Behaviors
- Explicit Expression
- Exponential Function
- Even function
- Function
- Function Notation
- Geometric Sequence
- Interval Notation
- Linear Function
- Odd function
- Range
- Recursive Formula
EVIDENCE OF LEARNING
By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Explain what it means when two curves \( y = f(x) \) and \( y = g(x) \) intersect.
- Define and use function notation, evaluate functions at any point in the domain, give general statements about how \( f(x) \) behaves at different regions in the domain (as \( x \) gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.
- Explain the difference and relationship between domain and range and find the domain and range of a function from a function equation, table or graph.
- Examine data (from a table, graph, or set of points) and determine if the data is a function and explain any conclusions that can be drawn.
- Write a function from a sequence or a sequence from a function.
- Explain how an arithmetic or geometric sequence is related to its algebraic function notation.
- Interpret \( x \) and \( y \) intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear or exponential function in terms of the context of the function.
- Find and/or interpret appropriate domains and ranges for authentic linear or exponential functions.
- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.
- Estimate the rate of change of a function from its graph at any point in its domain.
- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.
- Accurately graph a linear function by hand by identifying key features of the function such as the \( x \)- and \( y \)-intercepts and slope.
- Graph a linear or exponential function using technology.
- Sketch the graph of an exponential function accurately identifying \( x \)- and \( y \)-intercepts and asymptotes.
- Describe the end behavior of an exponential function (what happens as \( x \) goes to positive or negative infinity).
- Discuss and compare two different functions (linear and/or exponential) represented in different ways (tables, graphs or equations). Discussion and comparisons should include: identifying differences in rates of change, intercepts, and/or where each function is greater or less than the other.
- Write a function that describes a linear or exponential relationship between two quantities and combine different functions using addition, subtraction, multiplication, division and composition of functions to create a new function.
- Write recursively and an explicit formula for arithmetic and geometric sequences.
- Construct and compare linear and exponential models and solve problems. Recognize situations with a constant rate of change as well as those in which a quantity either grows or decays by a constant percent rate.